Kinematics of gravity-capillary waves under an evolving underwater current

International Ocean Vector Winds Science Team Meeting 2025

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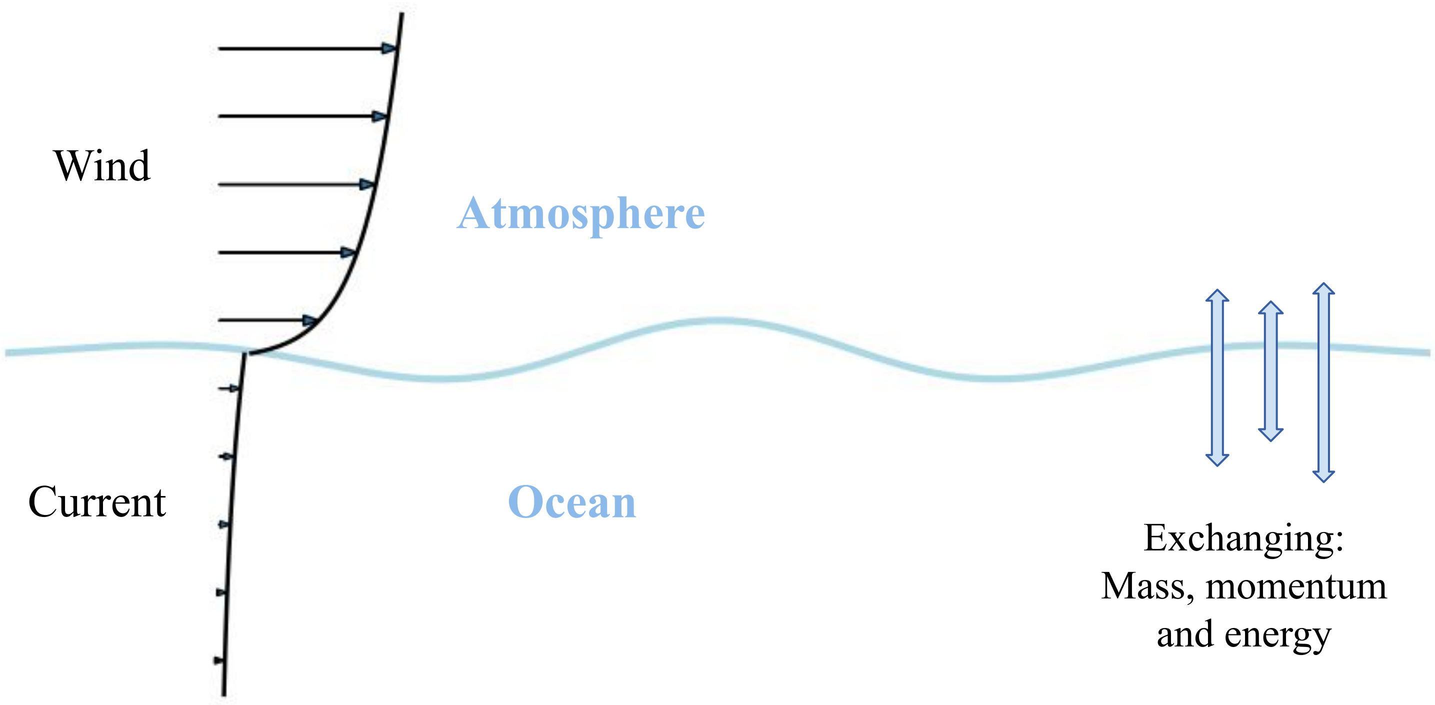
Introduction

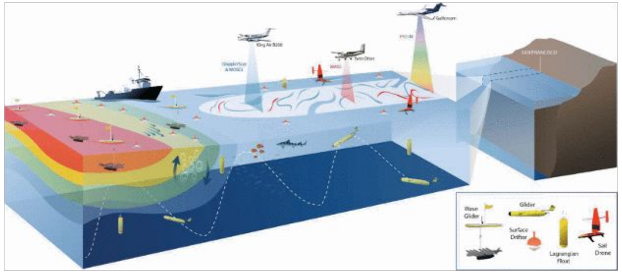
Waves **exist** at the **interface** between air and water → **Coupling** → **Oceanic** and **atmospheric Boundary Layers**



Motivation: Remote Sensing and Gravity-Capillary Waves

Remote Sensing

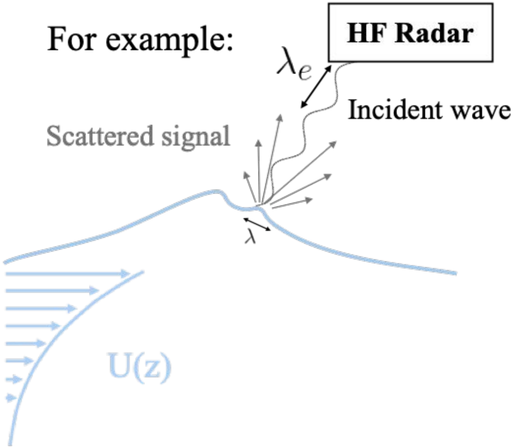
NASA's S-MODE Field Campaign



Farrar et al., 2021. Observations of Ocean Surface Currents.

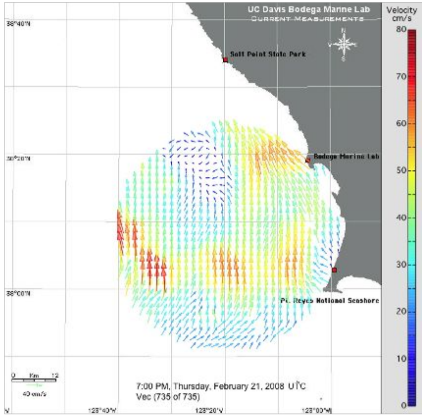
Studies submesoscale dynamics using these techniques

For example:



Measures surface currents using Bragg scattering

$$2\lambda sin(\theta) = \lambda_e$$



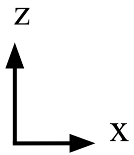
Bodega Marine Laboratory

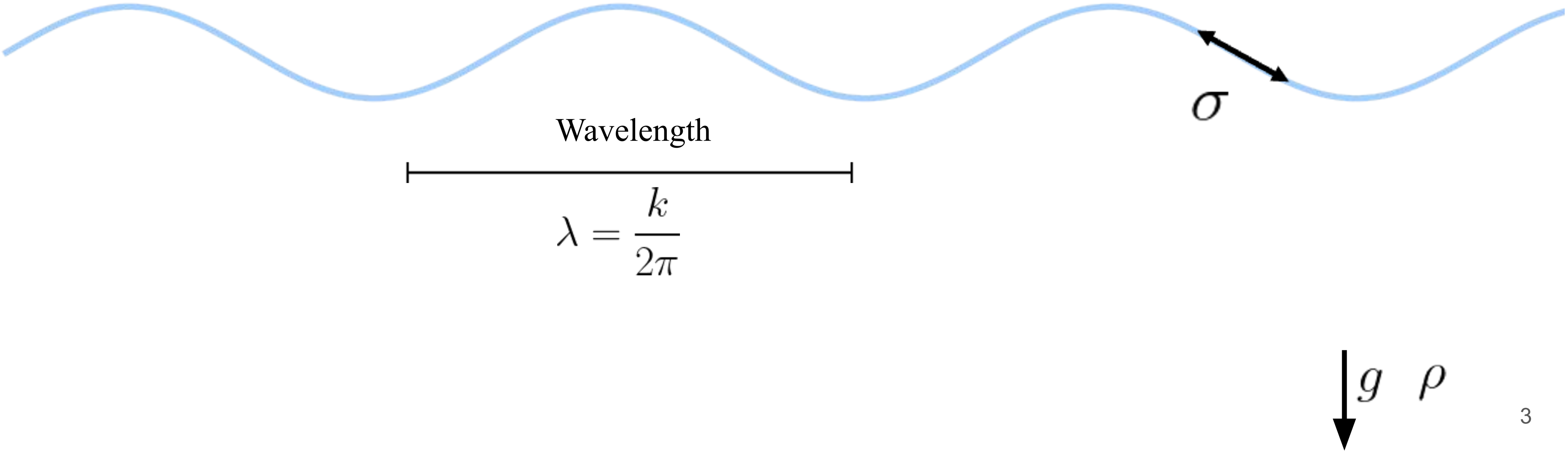
Some **Remote sensing** techniques are **sensitive to gravity-capillary** waves due to **Bragg scattering** and **surface roughness**.

Linear Dispersion Relation for **Gravity-Capillary waves**

Dispersion relation for **gravity-capillary waves**

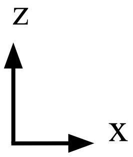
$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3}$$





How are currents affecting Gravity-Capillary waves?

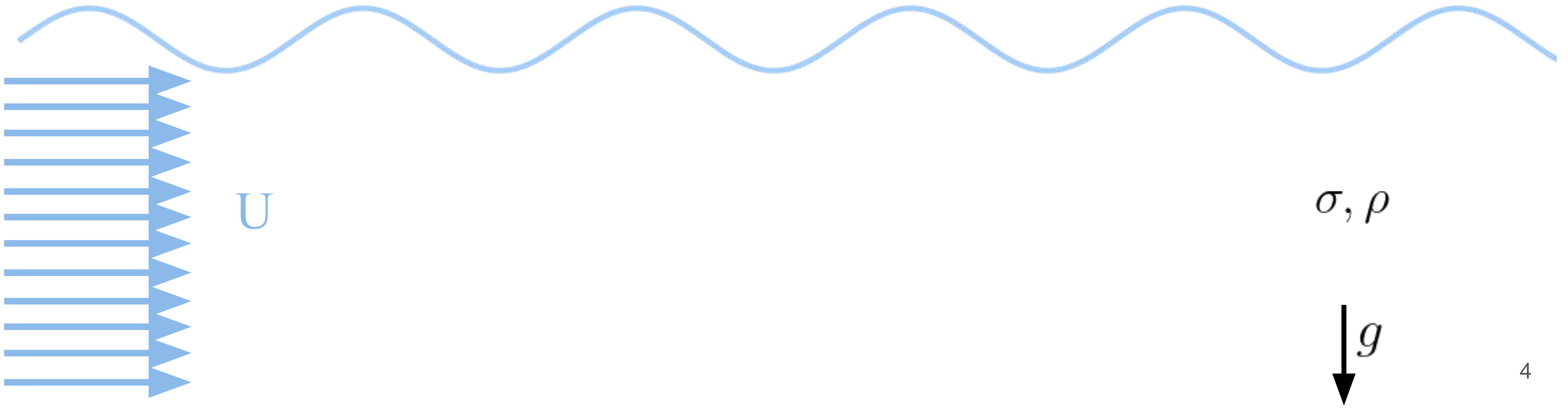
Let’s start considering a **constant velocity** U



$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3} + \mathbf{u} \cdot \mathbf{k}$$

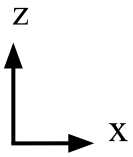
Doppler effect

When current flows in same direction as wave, **observed frequency increases**



Depth-Dependent Current Effects on waves

In reality, ocean currents vary with **depth**



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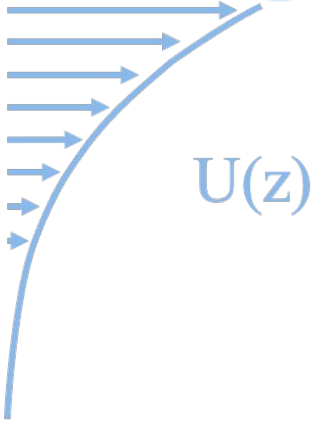
Doppler effect



Which velocity now that U depends on z?

Depth-Dependent Current Effects on waves

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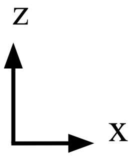
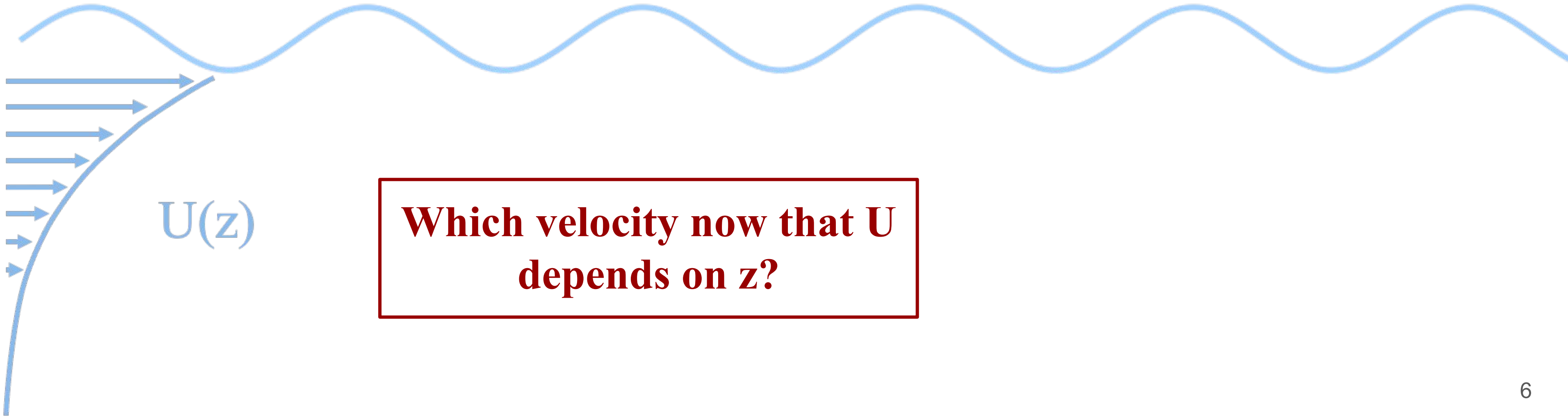


$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3} + u_{eff}(k)k$$

Doppler effect



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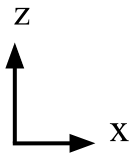
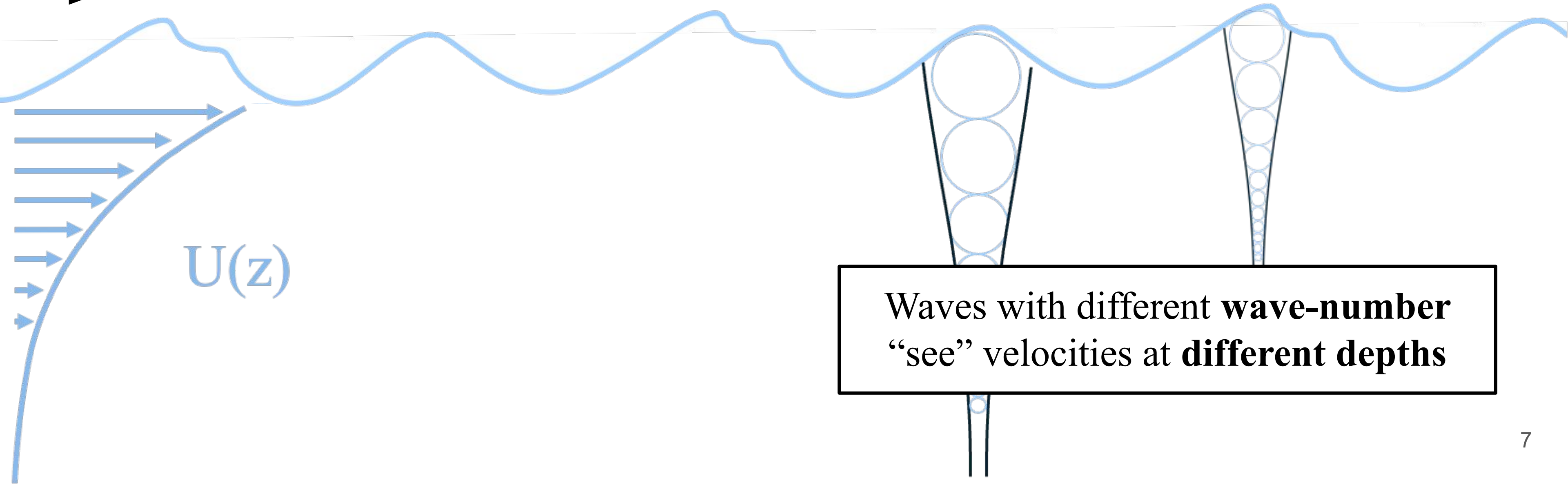


$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3} + u_{eff}(k)k$$

Doppler effect



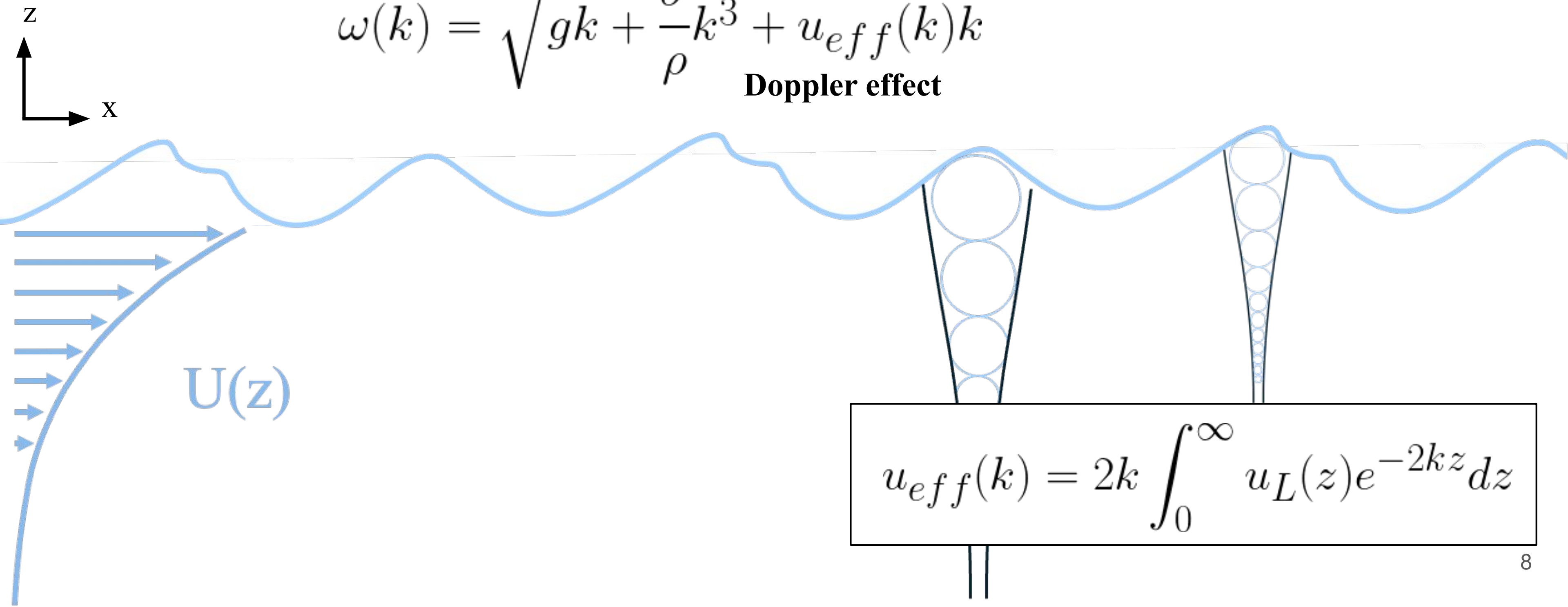
Waves with different **wave-number** “see” velocities at **different depths**

Depth-Dependent Current Effects on waves

In reality, ocean currents vary with **depth**

$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3} + u_{eff}(k)k$$

Doppler effect

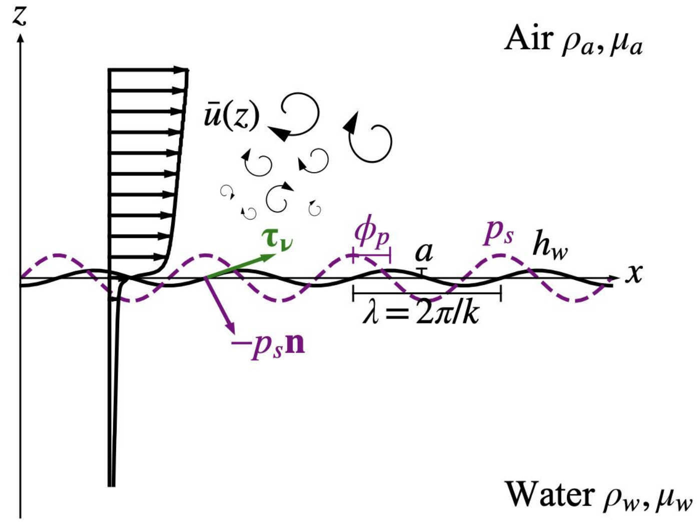


$$u_{eff}(k) = 2k\int_0^\infty u_L(z)e^{-2kz}dz$$

DNS Two-Phase Flow **Setup** (Based on Wu et al. 2022)

Waves forced by a turbulent boundary layer

$\tau_0 = \rho_a u_*^2 = H_a \partial_x p$



(Wu et al. 2022)

Several **non-dimensional** parameters

$u_*/c = 0.25, 0.5, 0.75$ Wind forcing

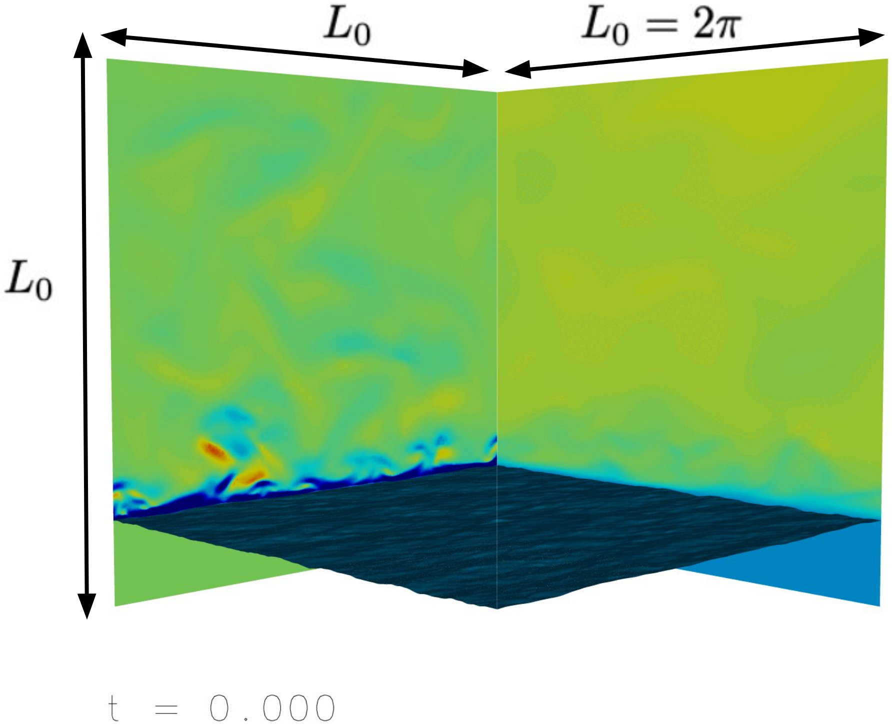
$Re_\tau = \dfrac{\rho_a u_* H_a}{\mu_a} = 720$ Turbulent air Reynolds number

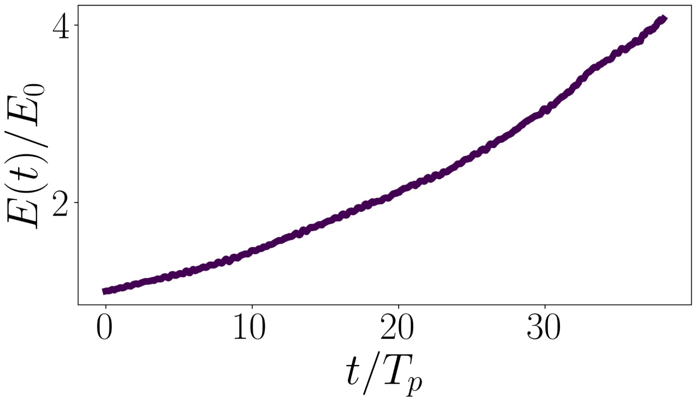
$k_p H_s = 0.08, 0.16$ Initial wave amplitude

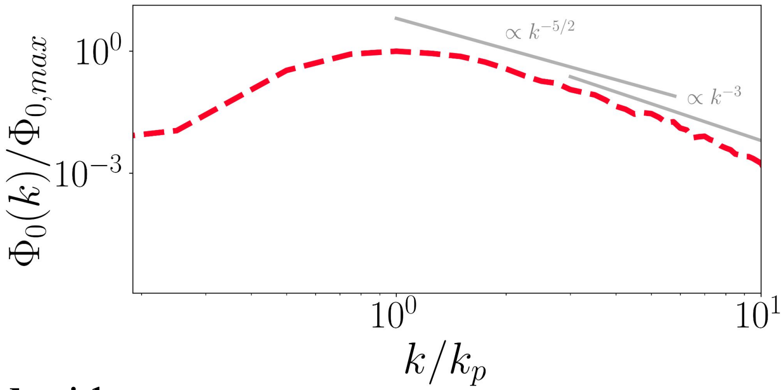
$Bo = \dfrac{(\rho_w - \rho_a) g}{k_p^2 \sigma} = 200$ Wave Bond number

$Re_w = \dfrac{\rho_w c \lambda_p}{\mu_w} = 720$ Wave Reynolds number

DNS Two-Phase Flow



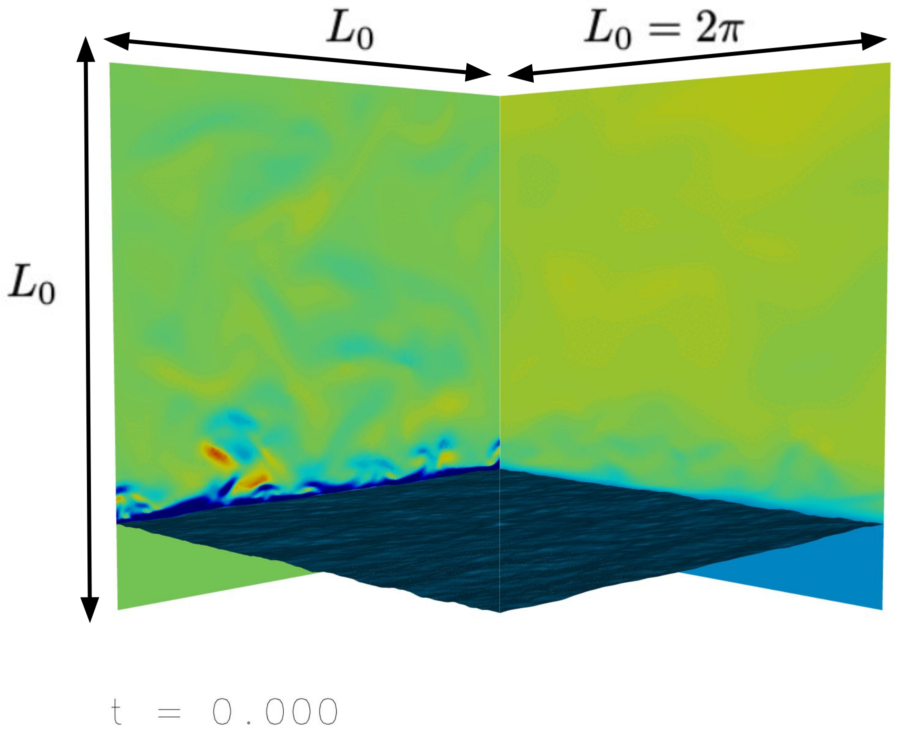


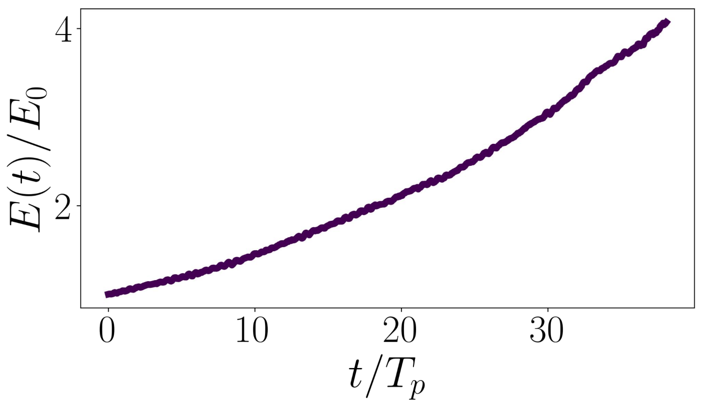


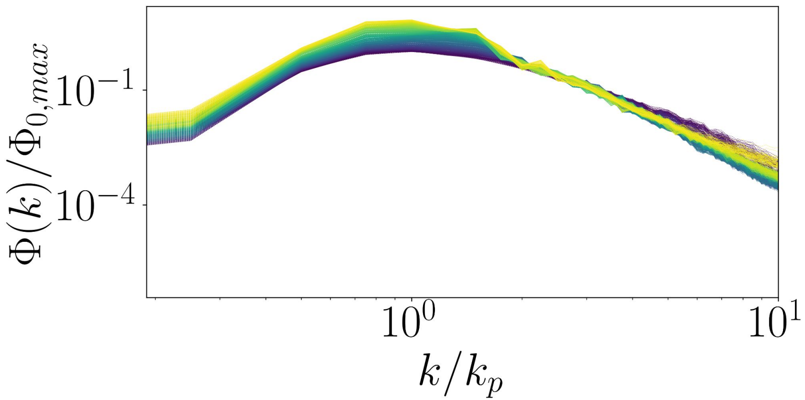
Temporally evolving **broadbanded wavefield, coupled** with **turbulent boundary layers** on both **air** and **water**

Basilisk: Open-source solver (developed by Stephane Popinet): **http://basilisk.fr/**

DNS Two-Phase Flow



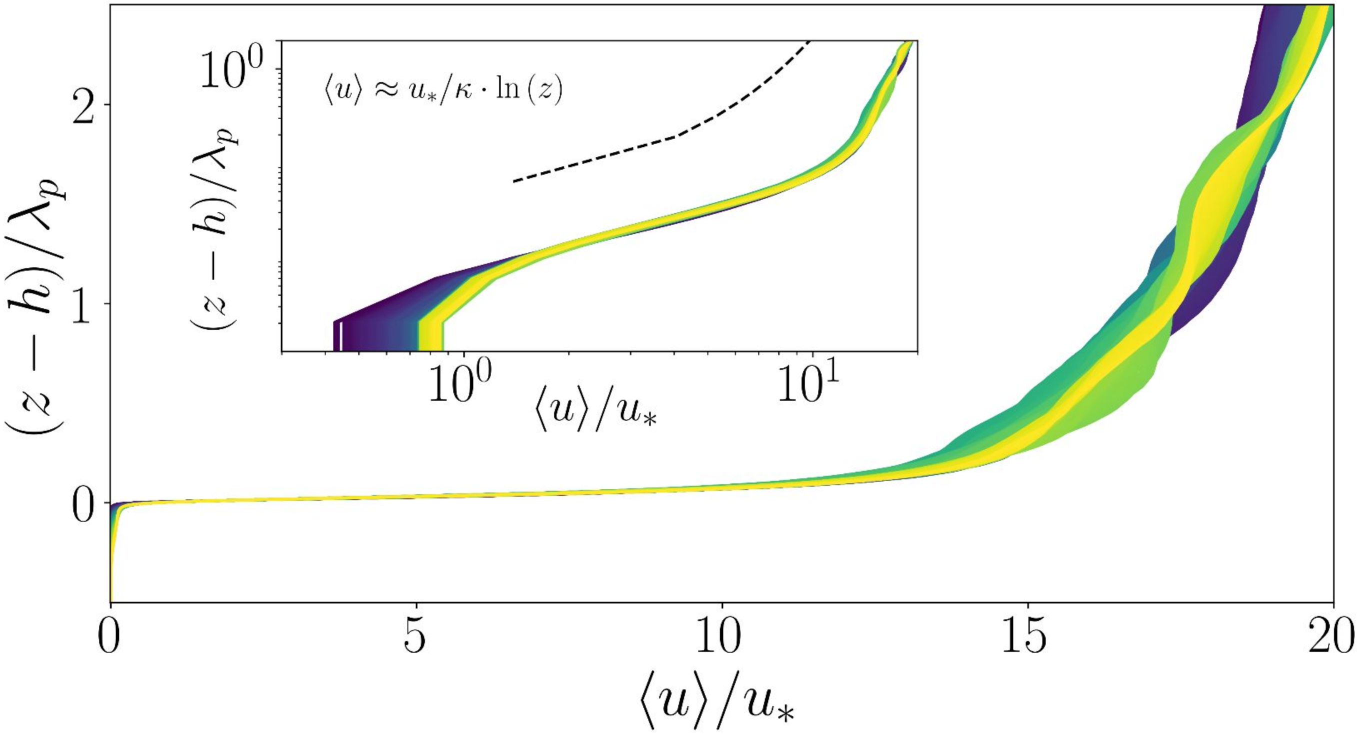




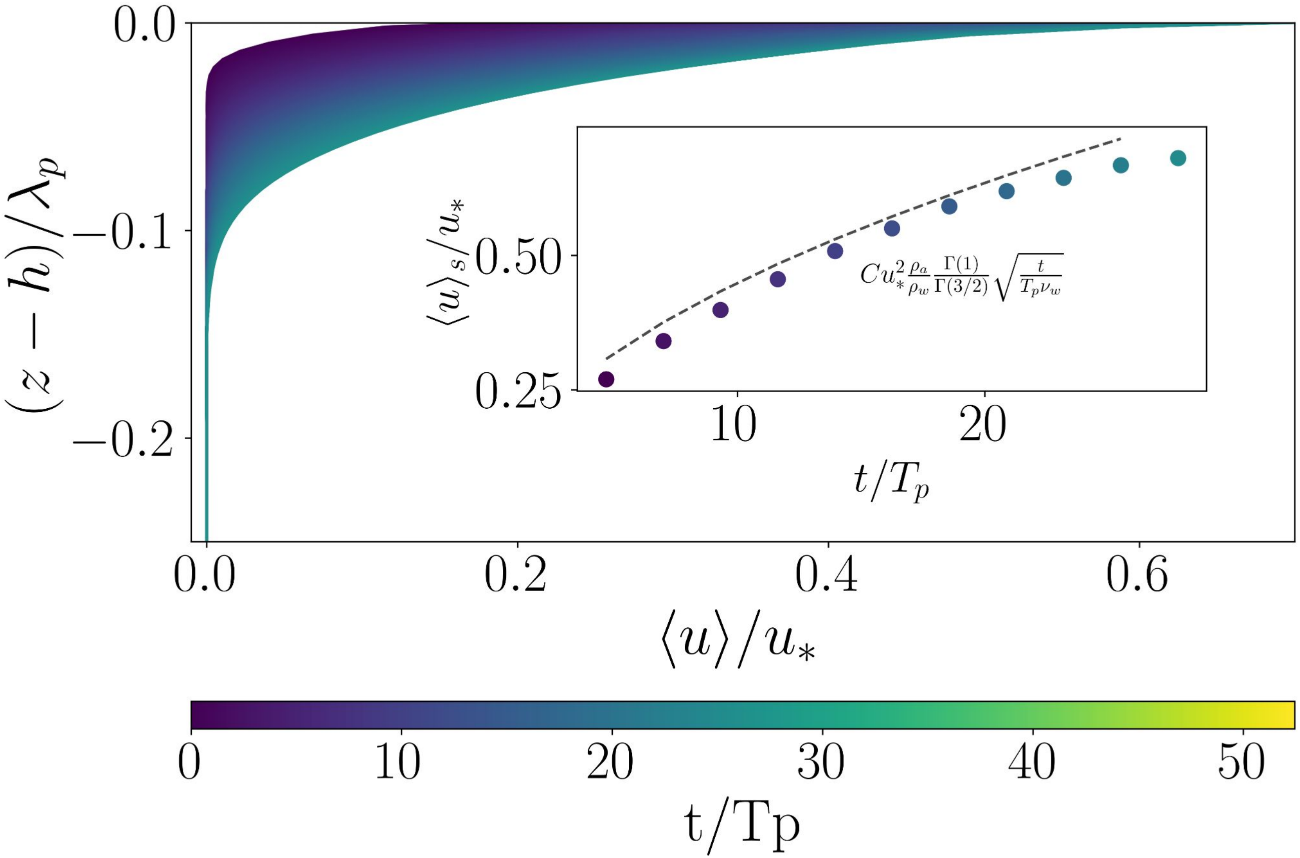
Temporally evolving **broadbanded wavefield, coupled with turbulent boundary layers** on both **air** and **water**

Basilisk: Open-source solver (developed by Stephane Popinet): **http://basilisk.fr/**

Mean flow: **Turbulent Boundary Layer** in **air**



Mean flow: **Developing viscous layer** in **water**

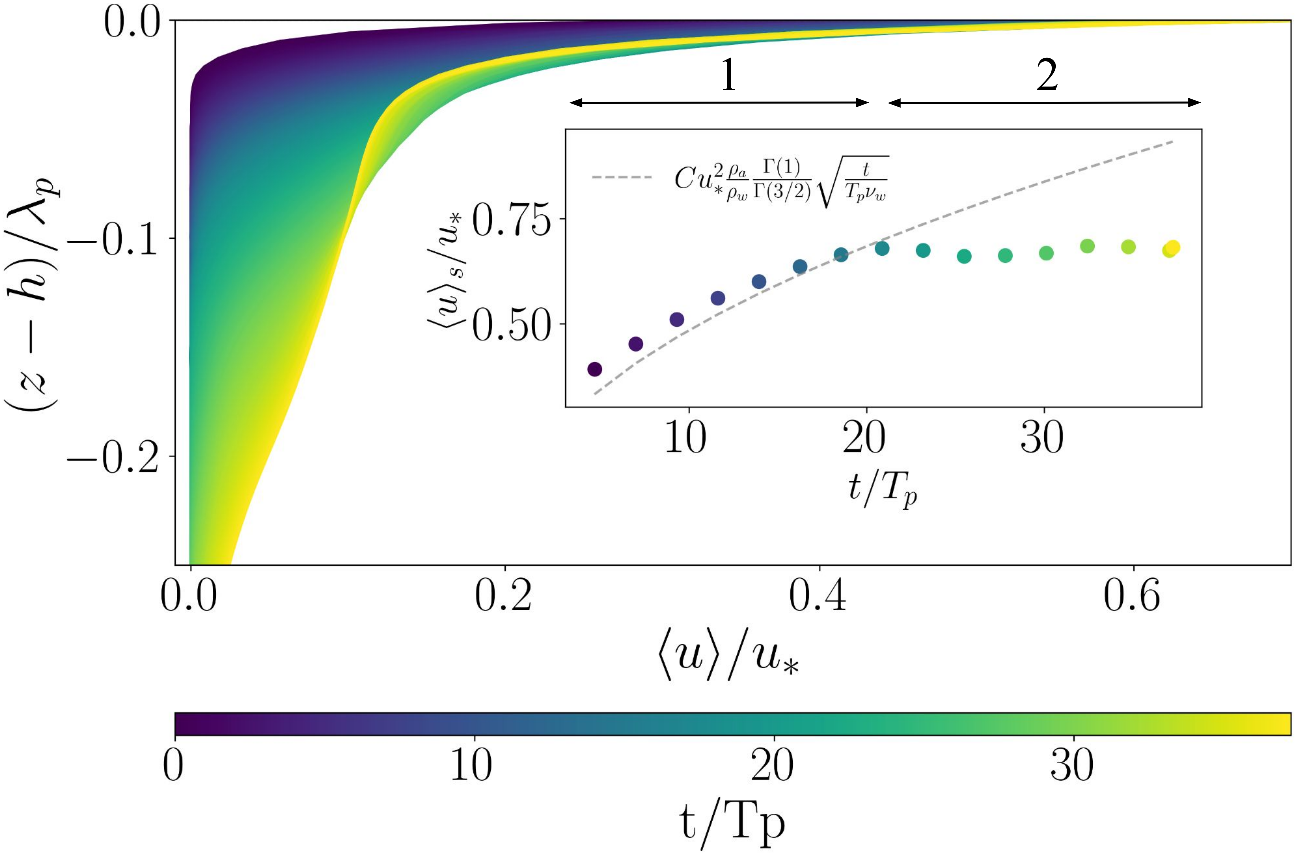


Developing viscous layer that will transition to a **turbulent boundary layer**

$$U_0(t) = \tau\frac{\Gamma(1)}{\Gamma(3/2)}\frac{\sqrt{\nu_w t}}{\mu_w} = u_*^2\frac{\rho_a}{\rho_w}\frac{\Gamma(1)}{\Gamma(3/2)}\sqrt{t/\nu_w}$$

(Veron & Melville, 2001)

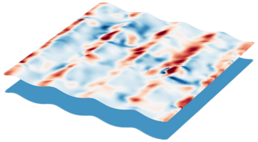
Mean flow: **Turbulent boundary layer** in **water**

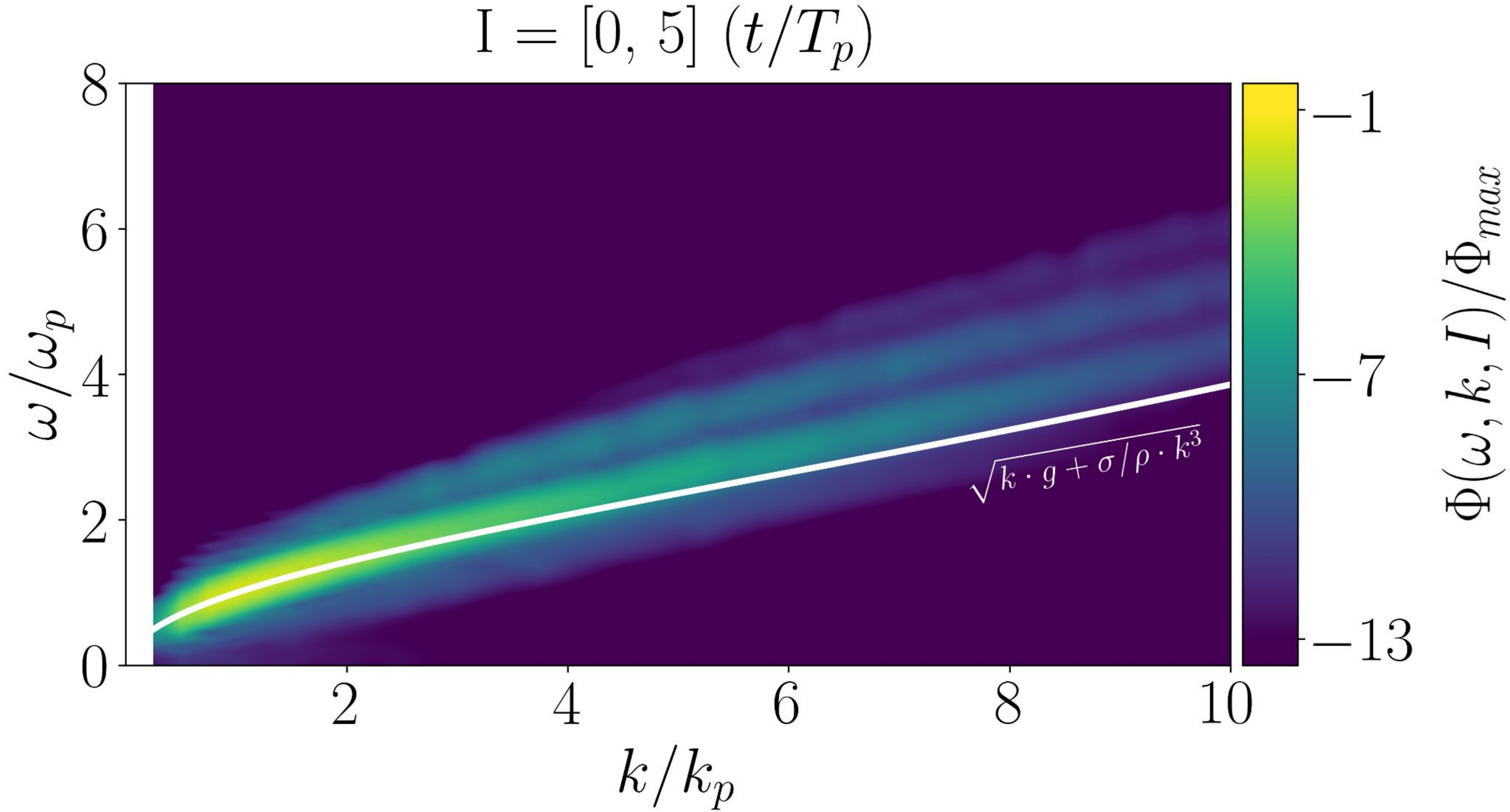


1. **Viscous** momentum diffusion
2. Fully **developed turbulence**

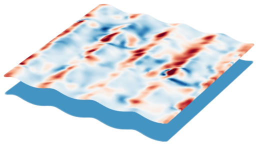
Now we want to look to the space-time spectrum and its evolution

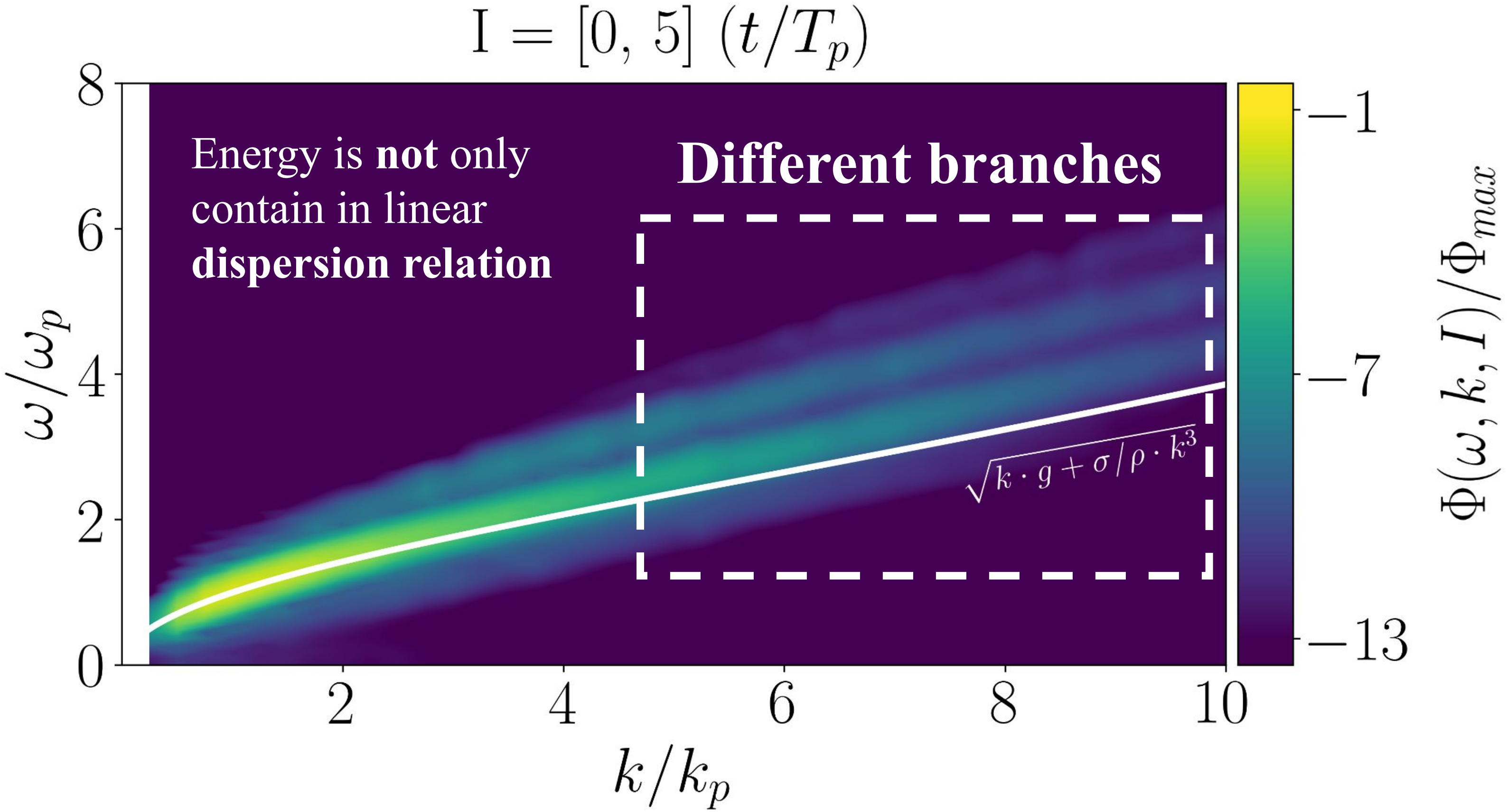
Spatio-temporal FFT analysis



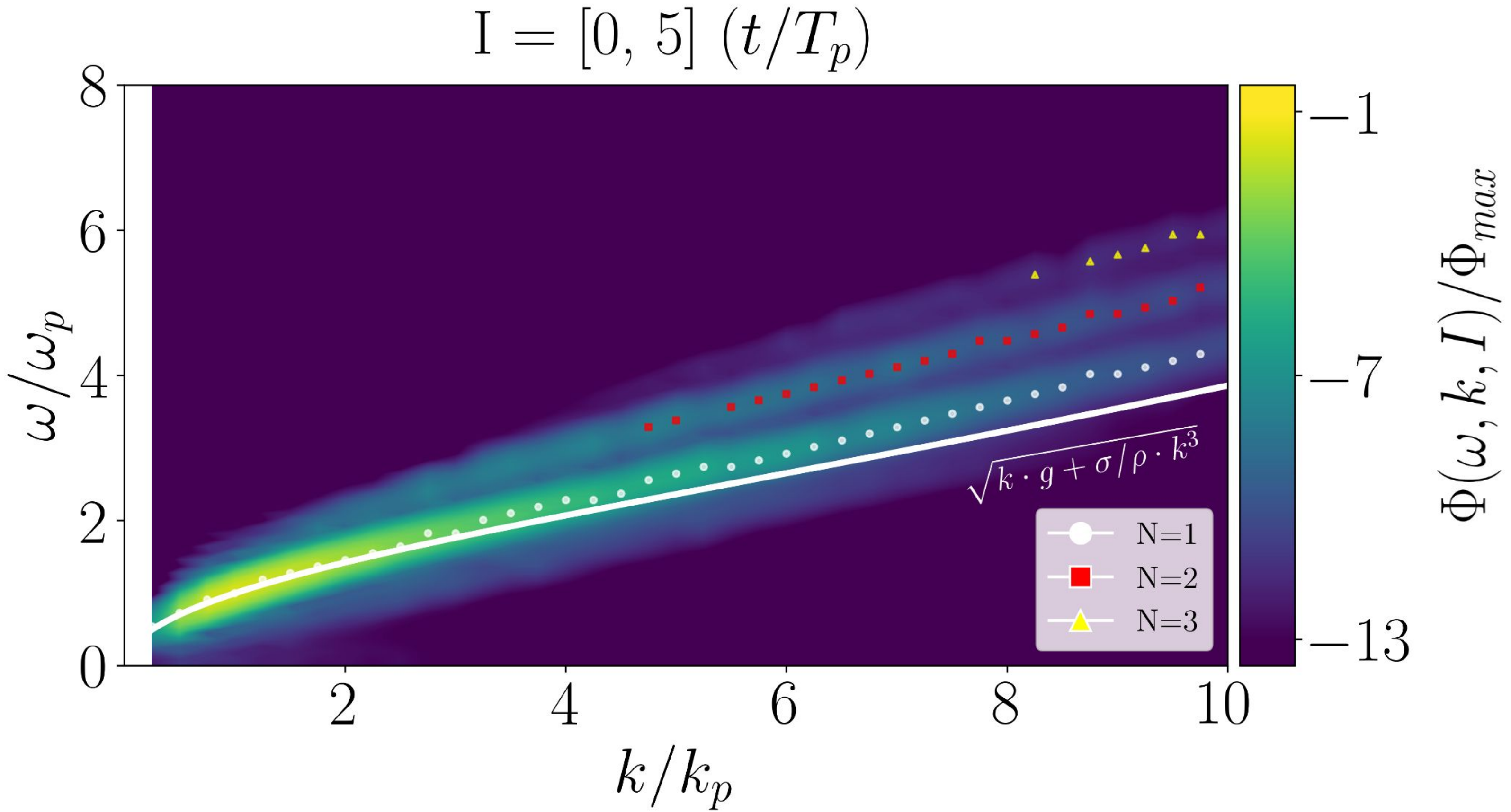


Spatio-temporal FFT analysis

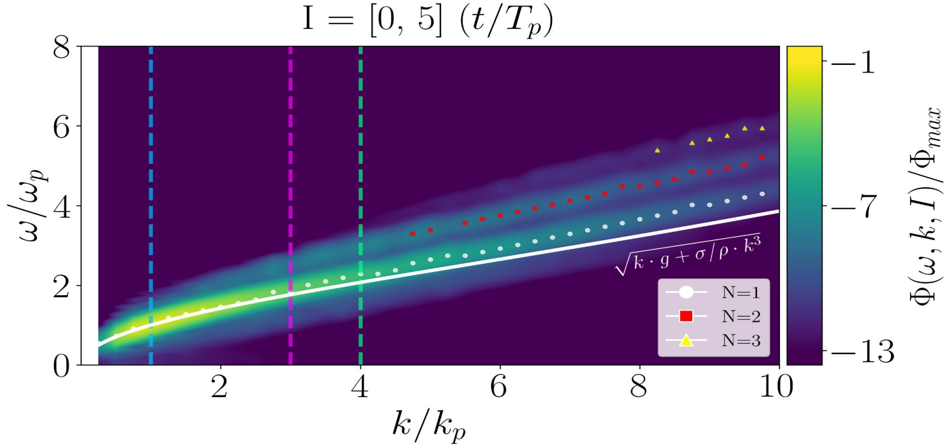


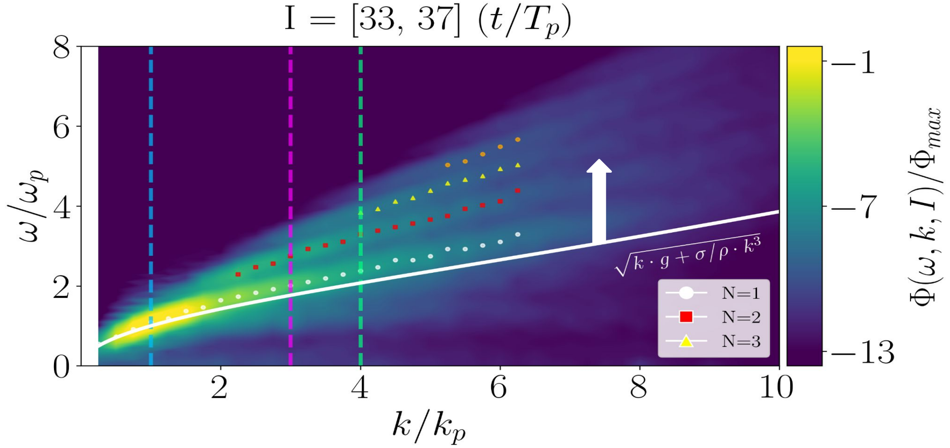


How do energy branches evolve in time?



Branches Time Evolution: Shift from Linear Dispersion Relationship





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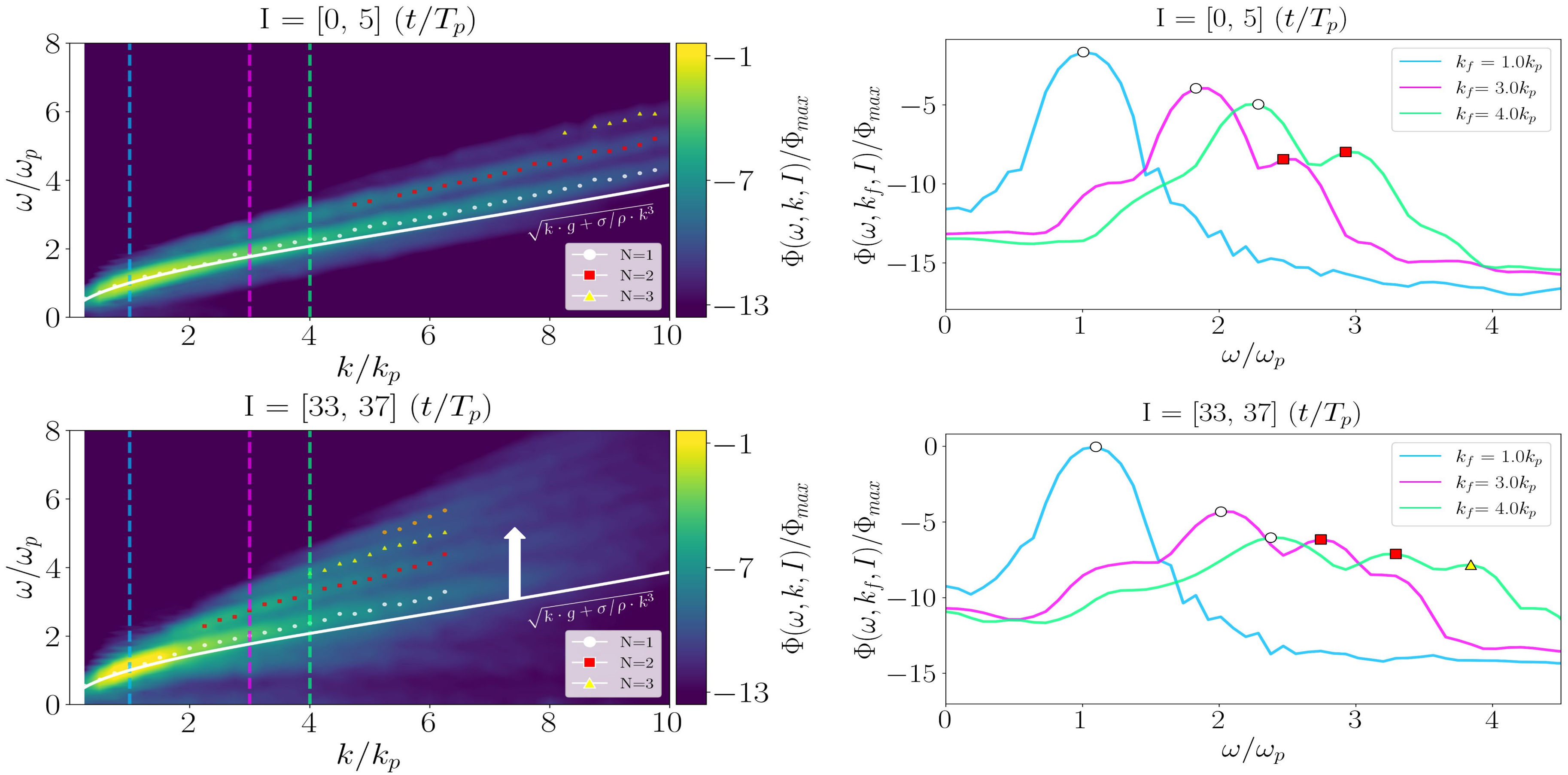
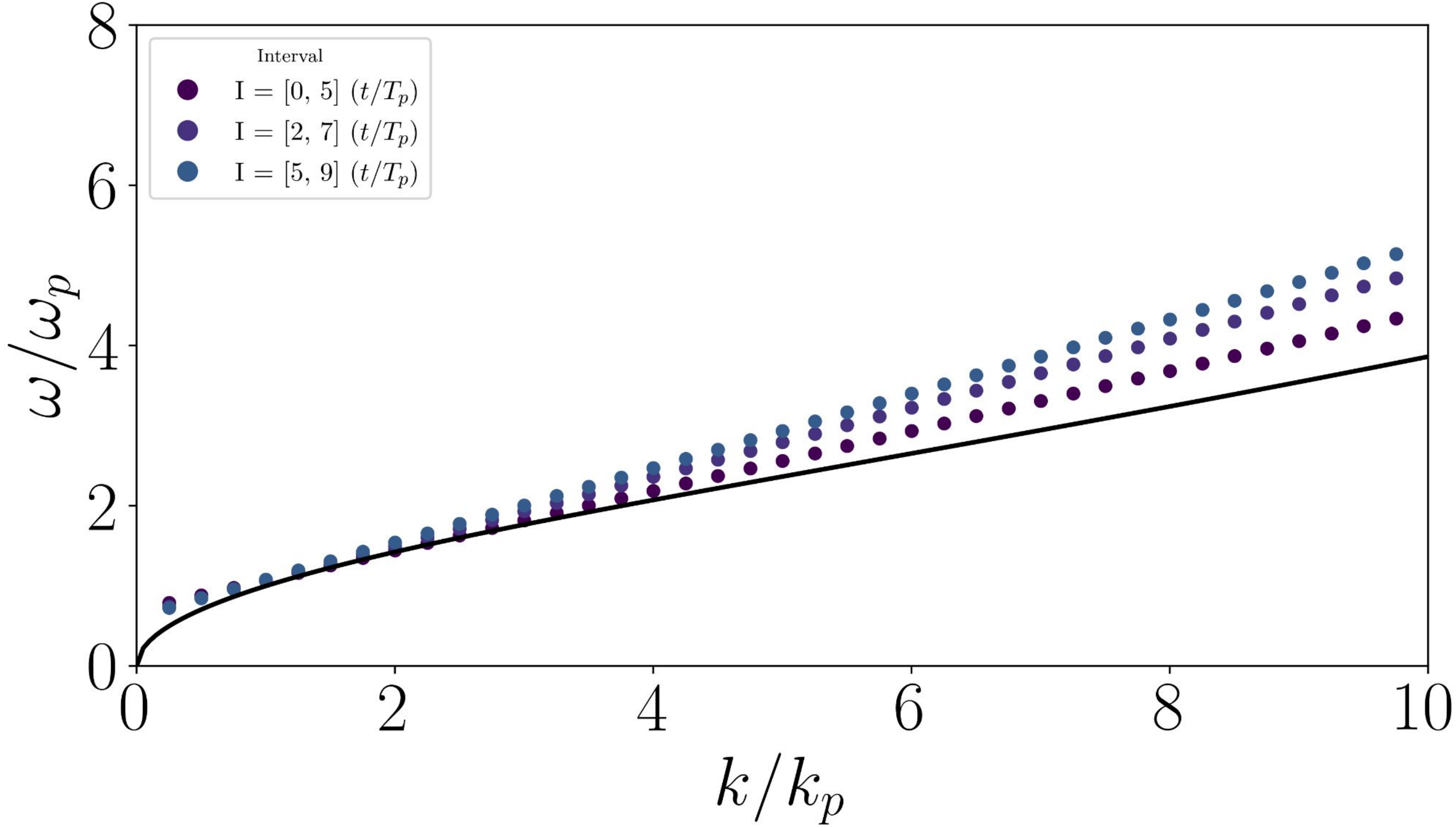
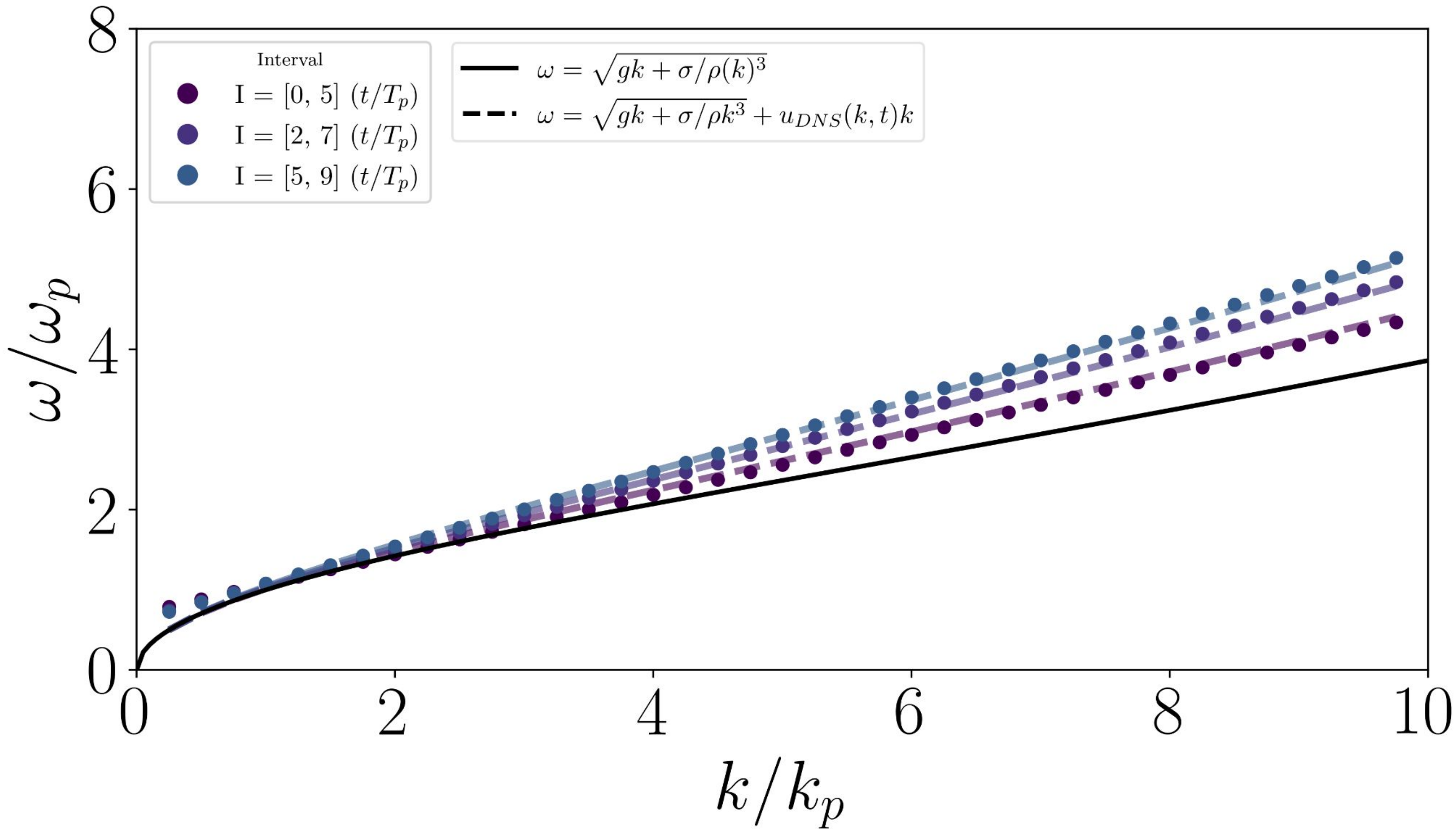


Branches Time Evolution: First Branch



Branches Time Evolution: Doppler shift first branch



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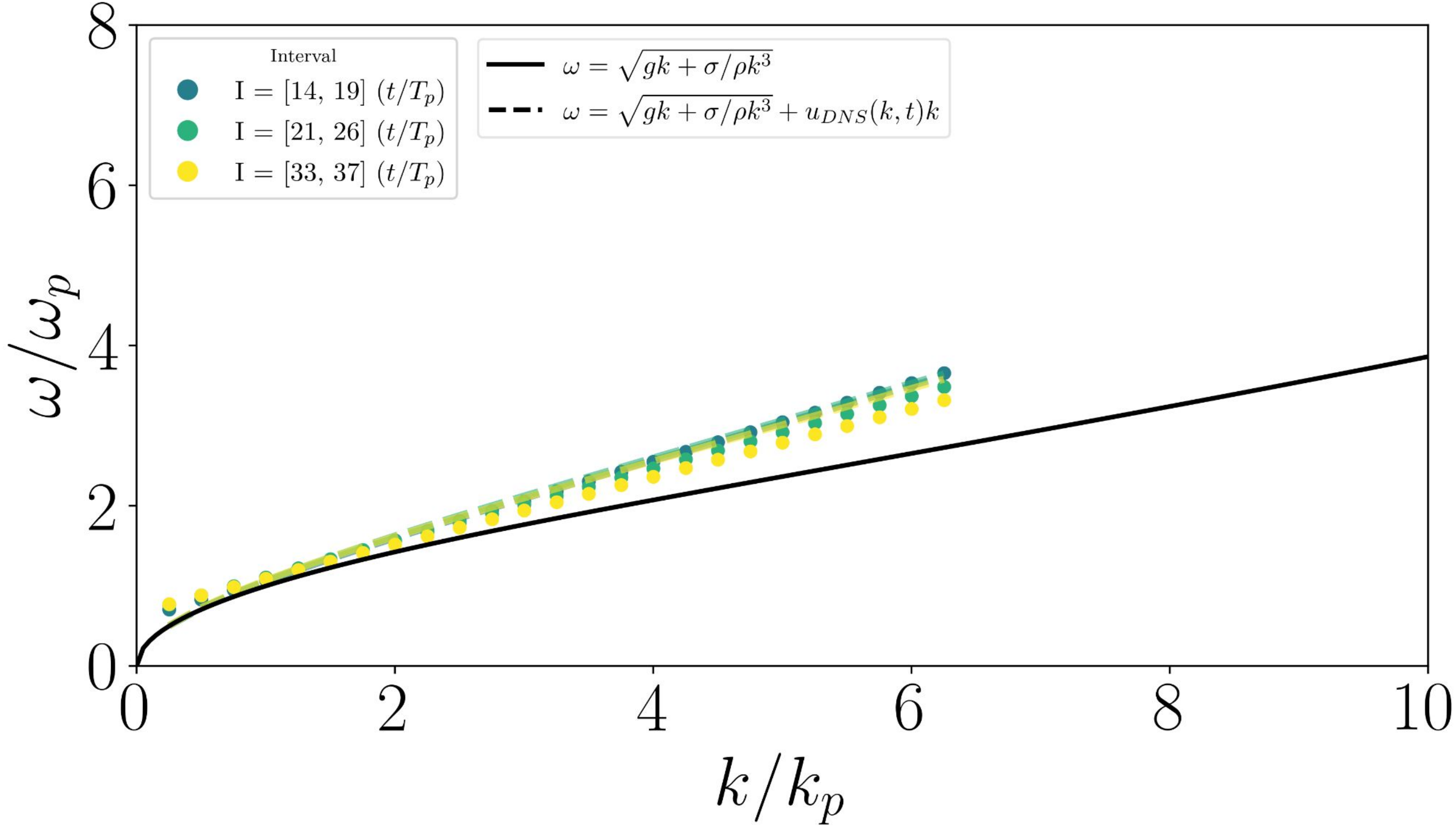

Branches Time Evolution: Higher harmonics

Primary mode

$$(k_*, \omega_*)$$

Branches Time Evolution: Higher harmonics

Primary mode

(k_*, ω_*)

Non-linear (non-resonant) interaction with itself →

Higher harmonics

$(2k_*, 2\omega_*)$

Branches Time Evolution: Higher harmonics

Primary mode

$$(k_*, \omega_*)$$

Non-linear (non-resonant) interaction with itself

→

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$$(2k_*, 2\omega_*)$$

$$(3k_*, 3\omega_*)$$

Branches Time Evolution: Higher harmonics

Primary mode

$$(k_*, \omega_*)$$

Non-linear (non-resonant) interaction with itself

Higher harmonics

$$(2k_*, 2\omega_*)$$

$$(3k_*, 3\omega_*)$$

$$k_N = N k_*$$

$$\Omega_N = N \omega_*$$

Branches Time Evolution: Higher harmonics

Primary mode

(k_*, ω_*)

Non-linear (non-resonant) interaction with itself →

Higher harmonics

$(2k_*, 2\omega_*)$

$(3k_*, 3\omega_*)$

$k_N = Nk_*$

$\Omega_N = N\omega_*$

$$\Omega_N(k_N) = N\underbrace{\sqrt{gk_N/N + \frac{\sigma}{\rho}(k_N/N)^3}}_{\omega(k_N/N)}$$

Branches Time Evolution: Higher harmonics (NLDR)

Primary mode

(k_*, ω_*)

Non-linear (non-resonant) interaction with itself →

Higher harmonics

$(2k_*, 2\omega_*)$

$(3k_*, 3\omega_*)$

↓

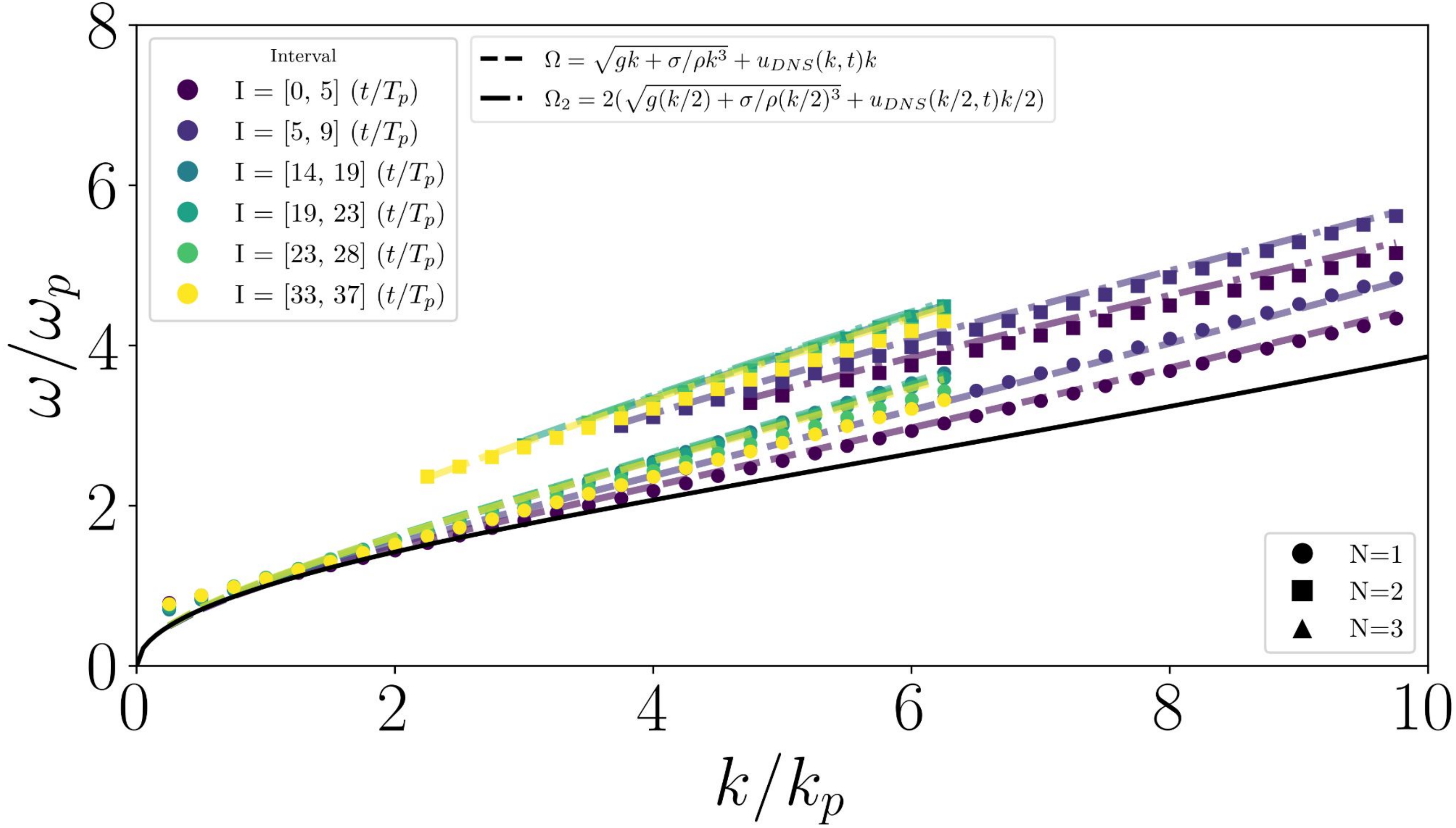
$k_N = Nk_*$

$\Omega_N = N\omega_*$

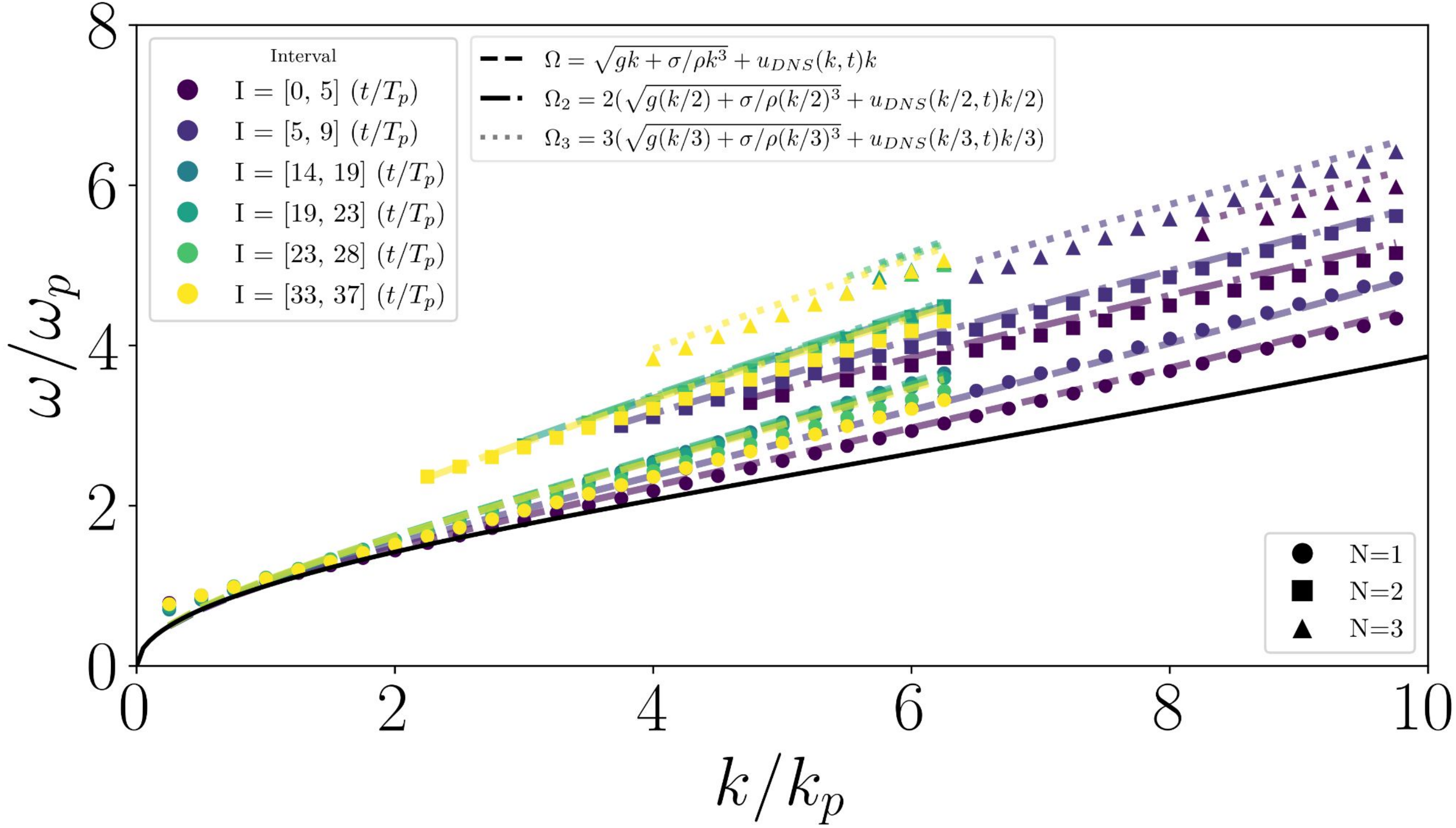
$$\Omega_N(k_N) = N\sqrt{gk_N/N + \frac{\sigma}{\rho}(k_N/N)^3}$$

$$\Omega_N(k_N, t) = N[\sqrt{gk_N/N + \frac{\sigma}{\rho}(k_N/N)^3}+u_{eff}(k_N/N, t)k_N/N]$$

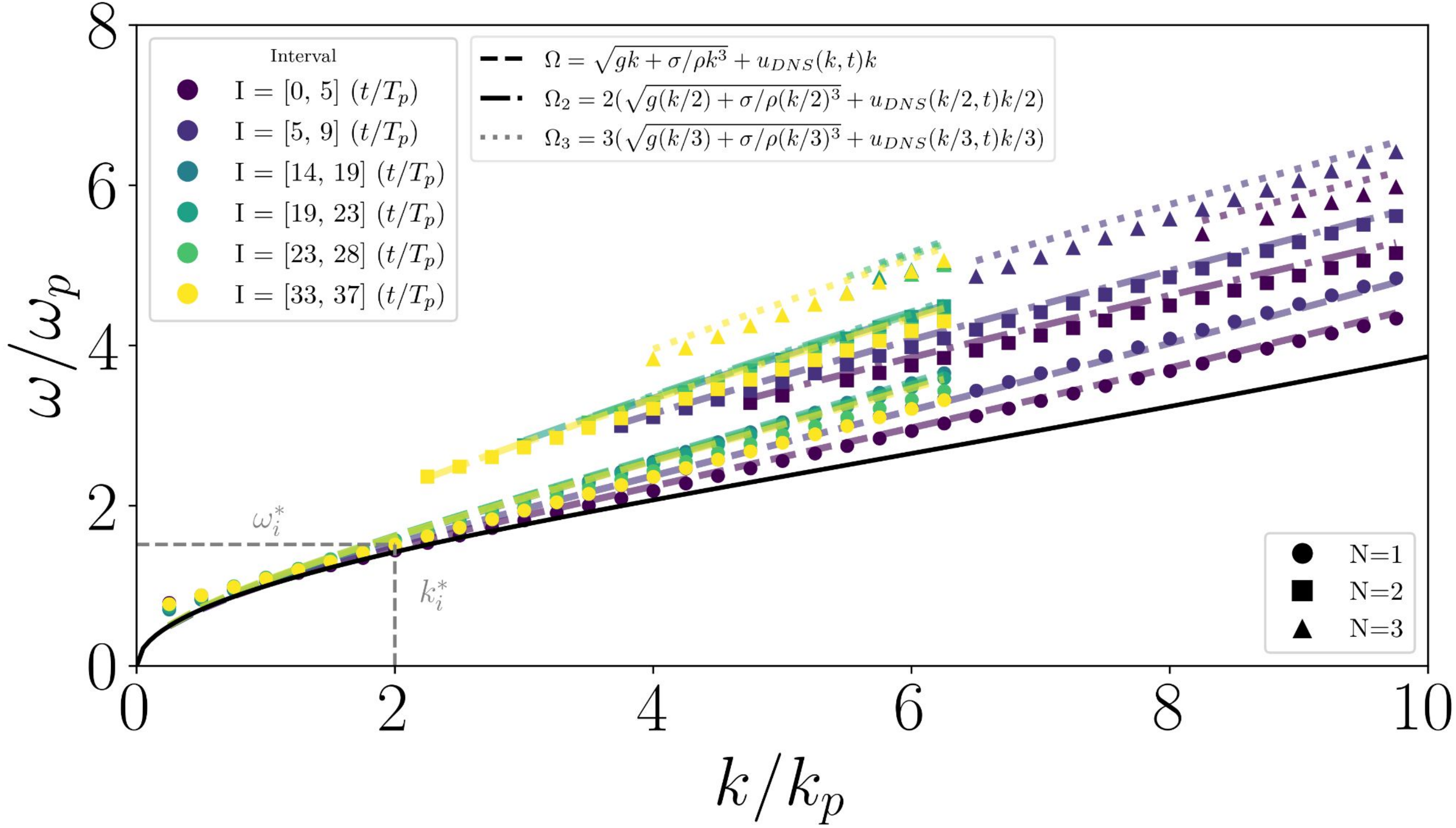
Branches Time Evolution: NLDR second branch



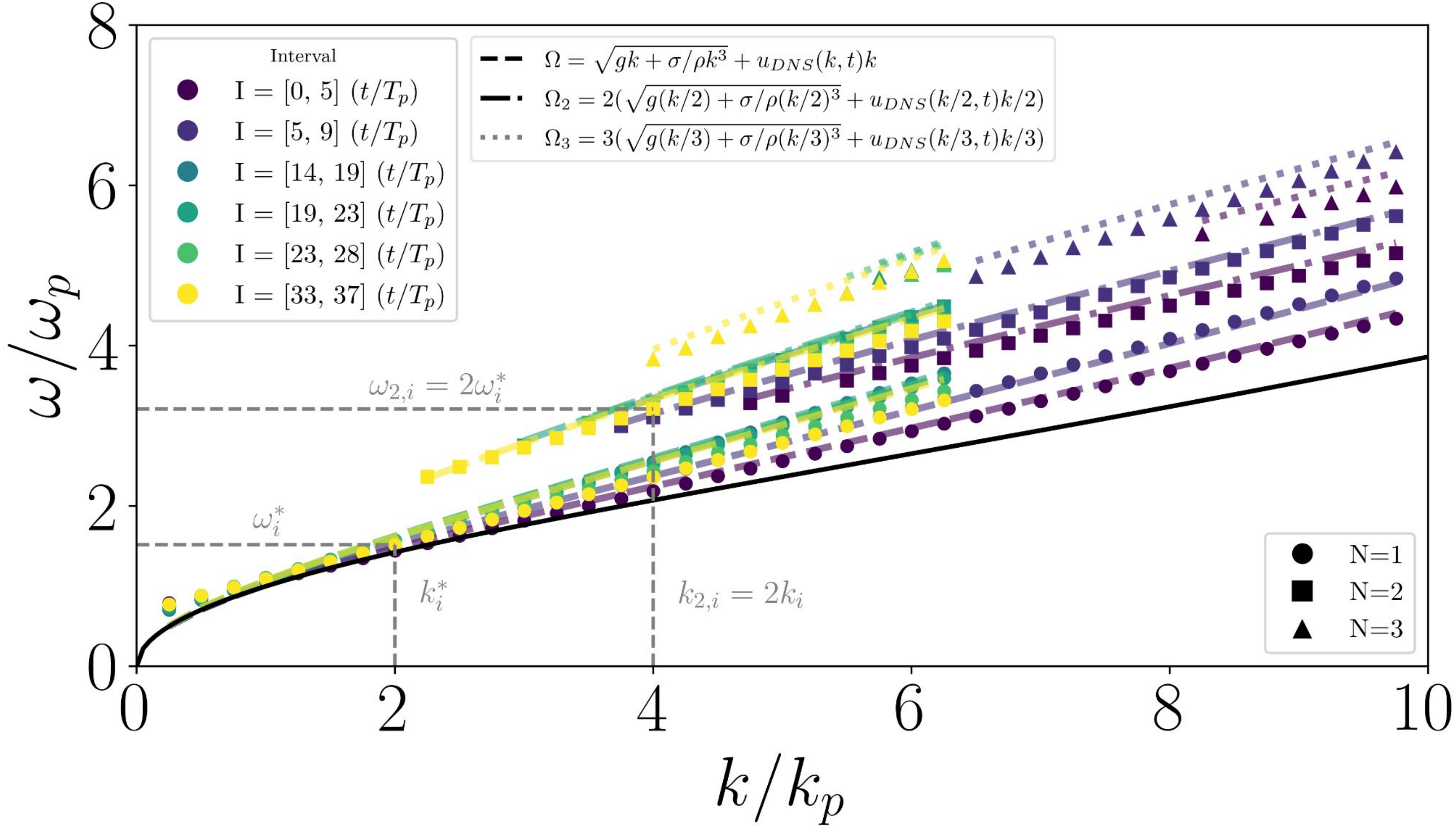
Branches Time Evolution: NLDR all branches



Branches Time Evolution: phase speed higher modes



Branches Time Evolution: what's the speed of higher modes?



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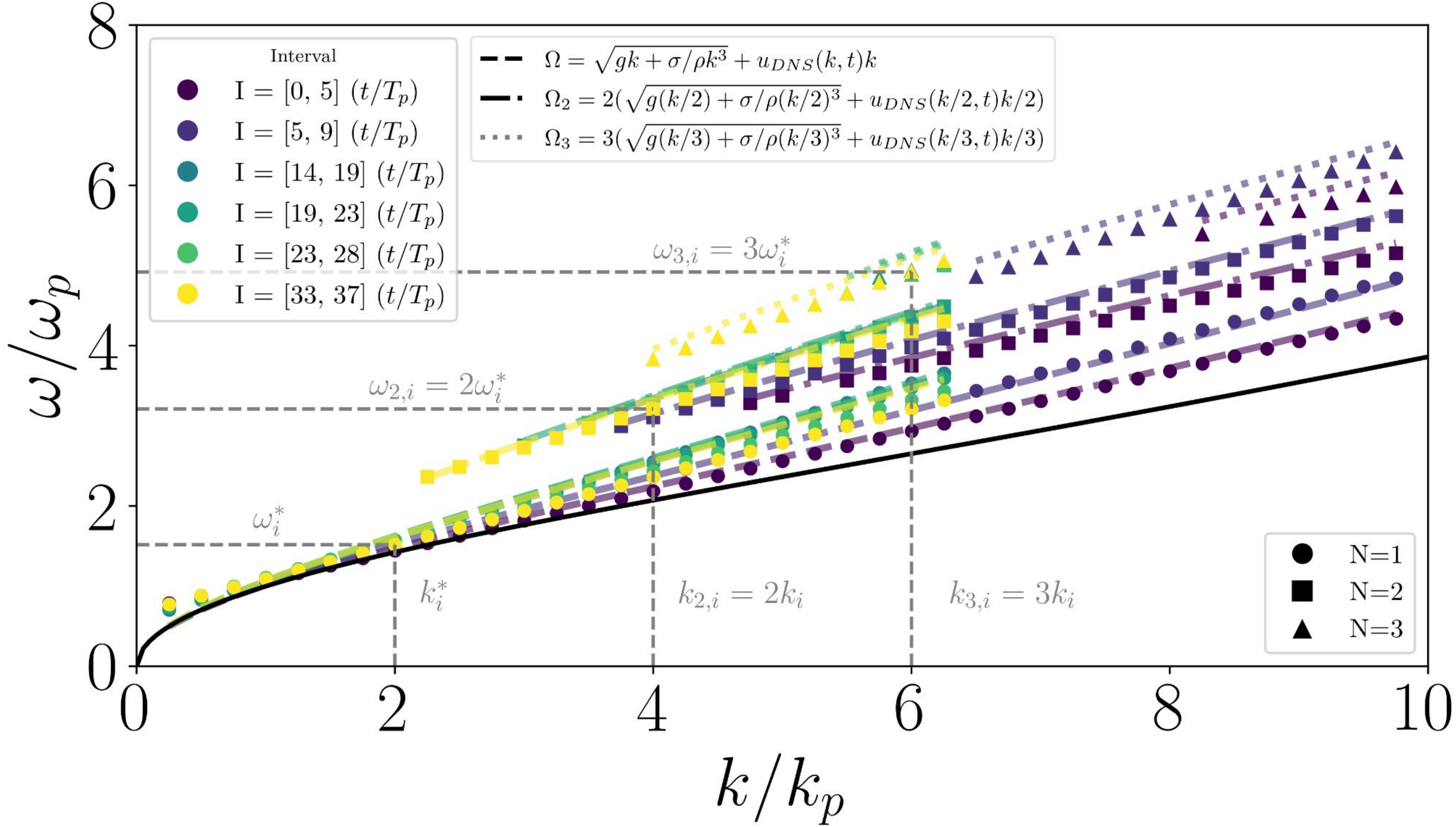
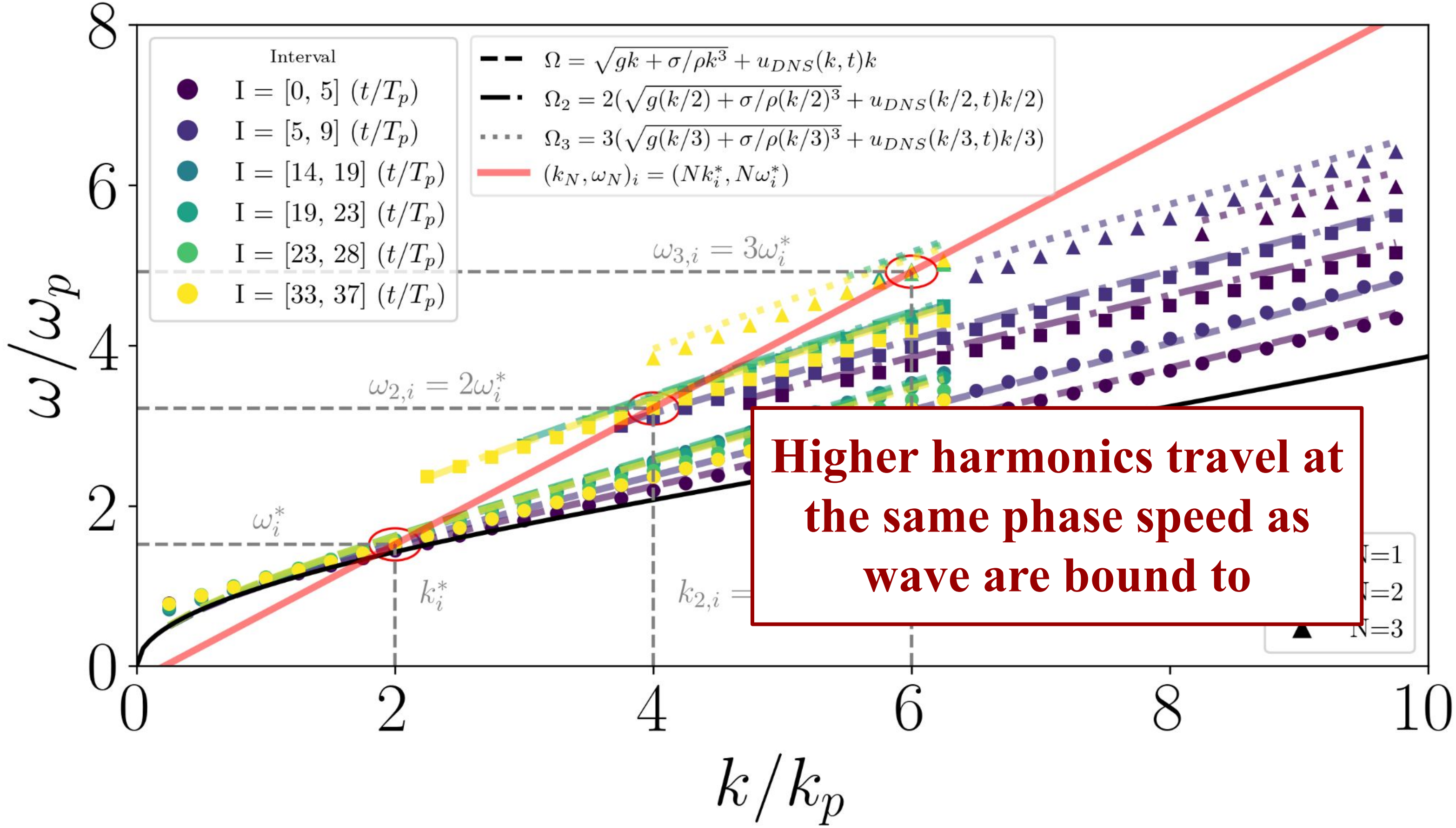
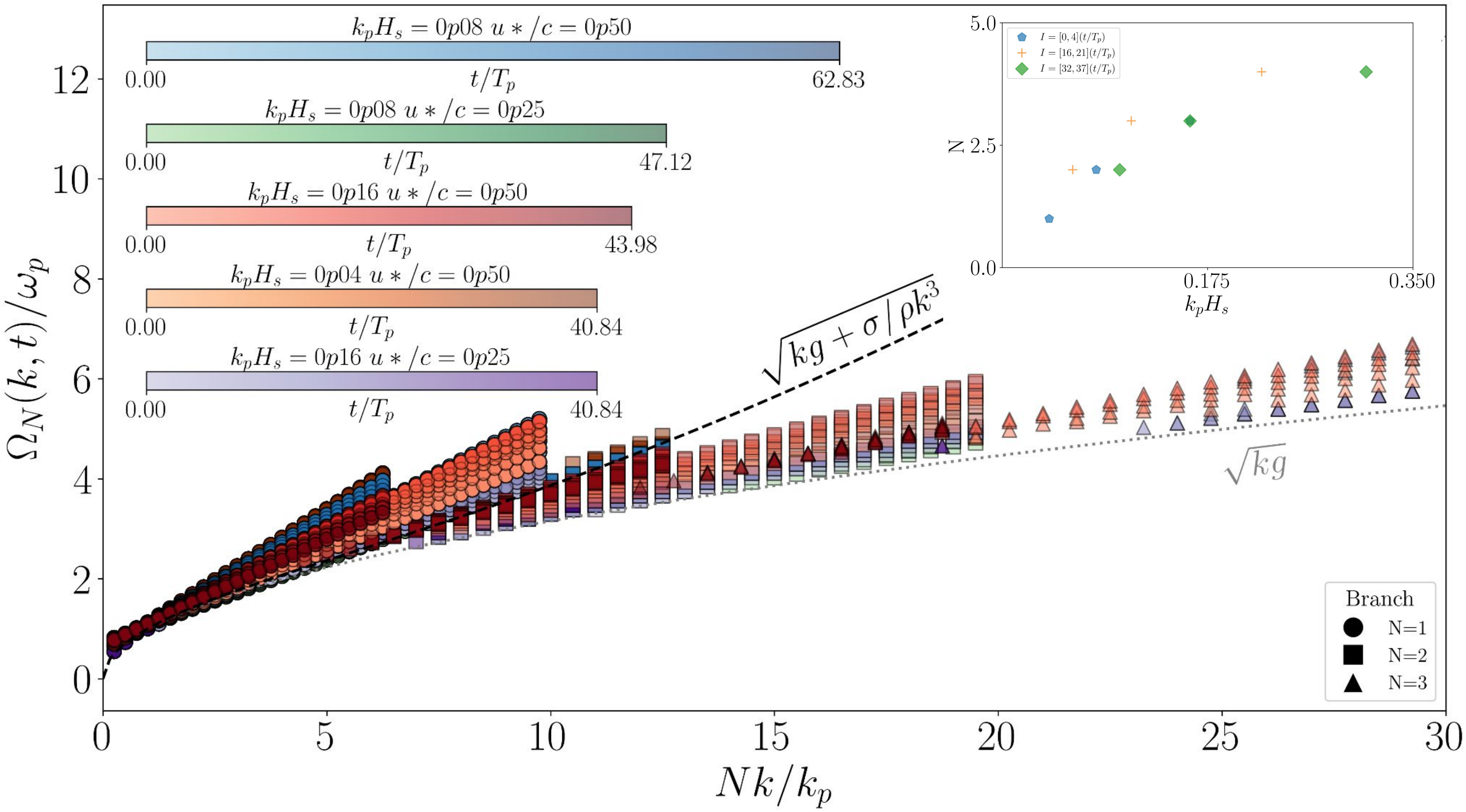


Branches Time Evolution: higher modes travel same phase speed that primary one



Does the nonlinear dispersion relation hold across different wind and wave initial conditions?

Validation of **dispersion relation** and **depth dependent velocity** over **different initial conditions**



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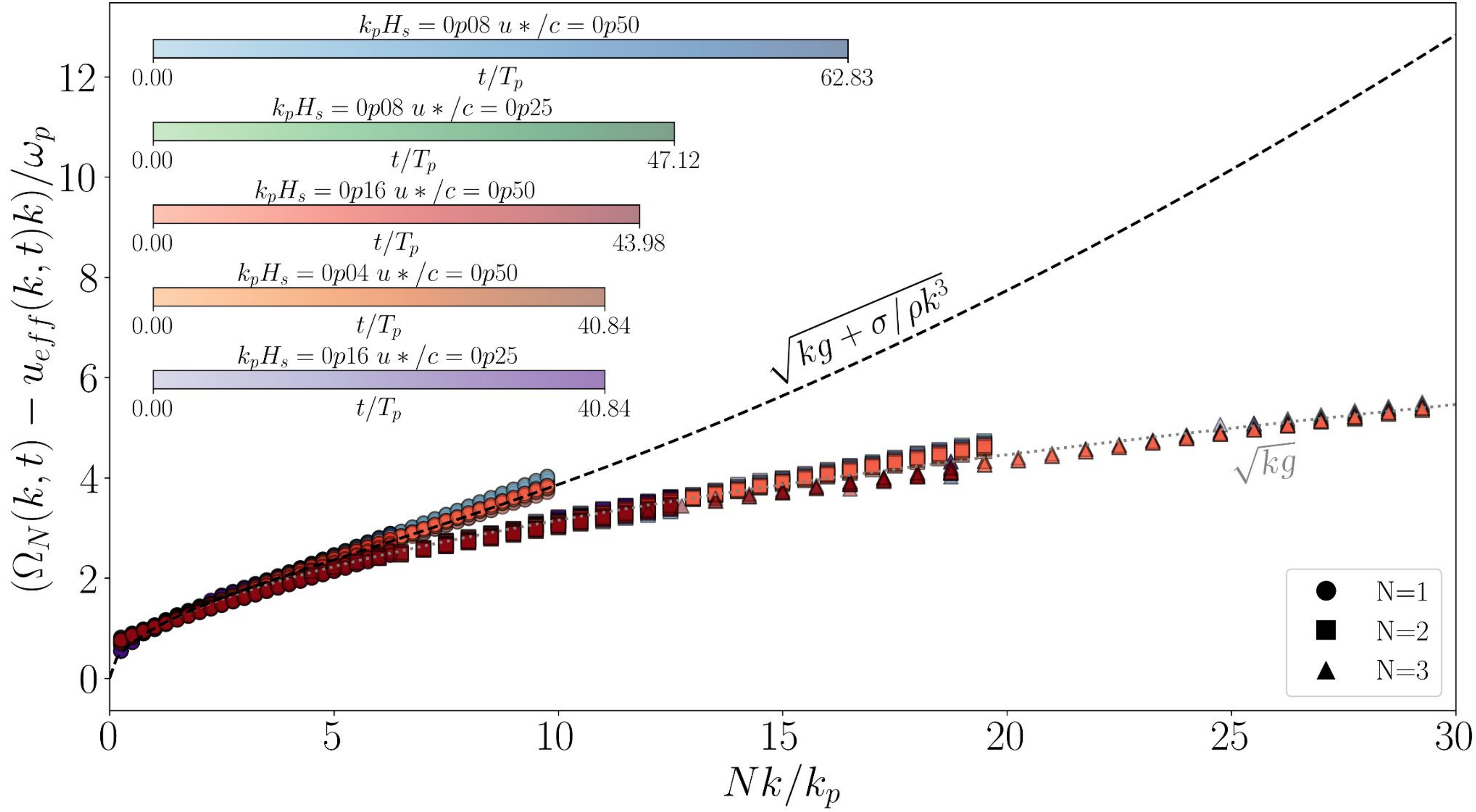


Conclusions

- We performed **DNS** of **fully coupled wind-forced broad-banded wave** fields using the open-source solver **Basilisk**, extending the work of Wu, Popinet, and Deike (2022, JFM).
- The flow is characterized by **a turbulent boundary layer** on the **air side** and a **viscous self-similar boundary layer** in the **water**, which transitions to a **turbulent state**.
- **Analysis** of the **evolution** of the **wavenumber-frequency** wave spectra **in time**:
  - **We validate a Non Linear Equation that accounts for the different harmonics:\.**
  - **Doppler** shift is characterized with a using a **weighted average depth-varying velocity** integrated from the **DNS**.

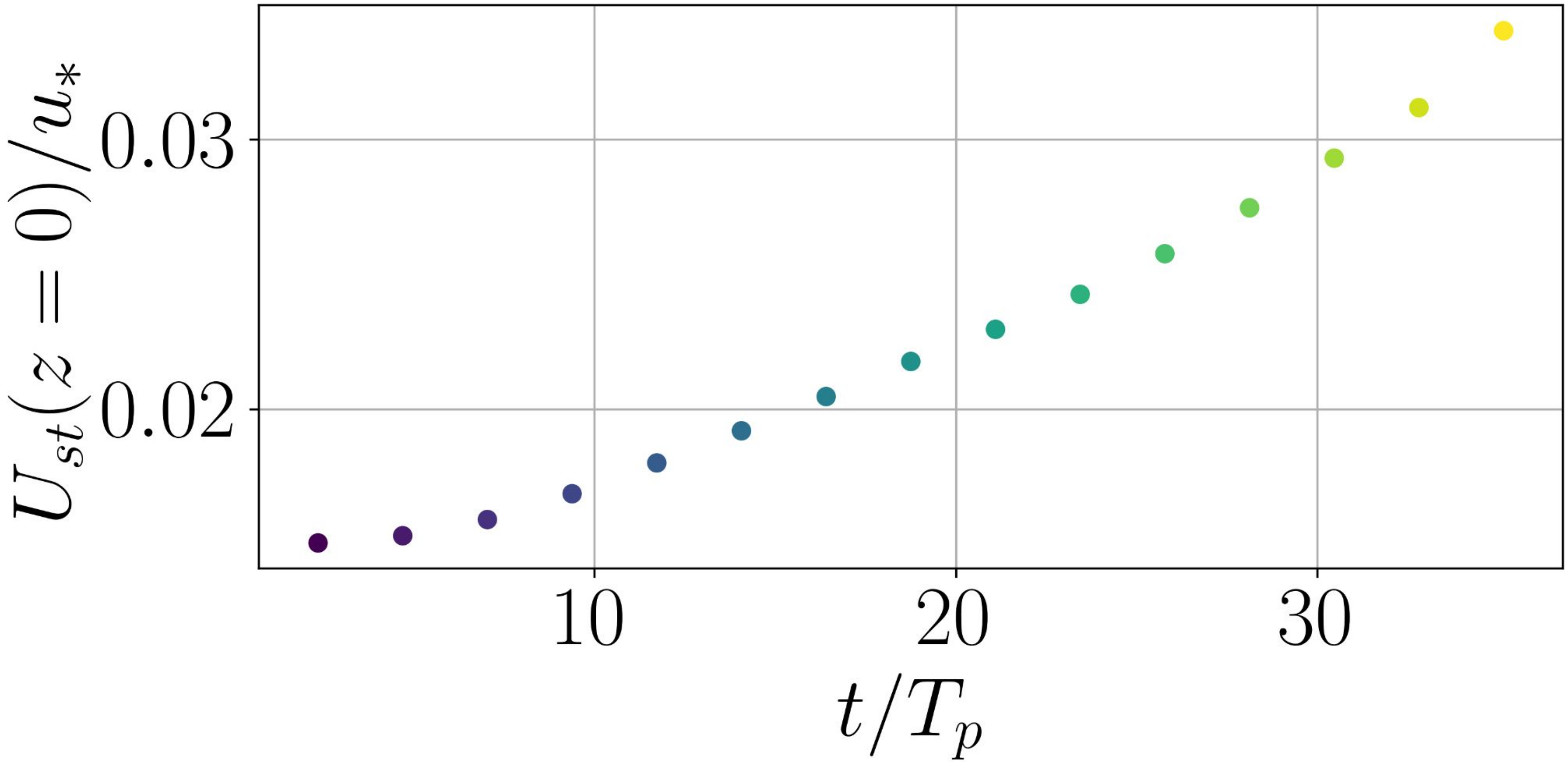
PRINCETON UNIVERSITY

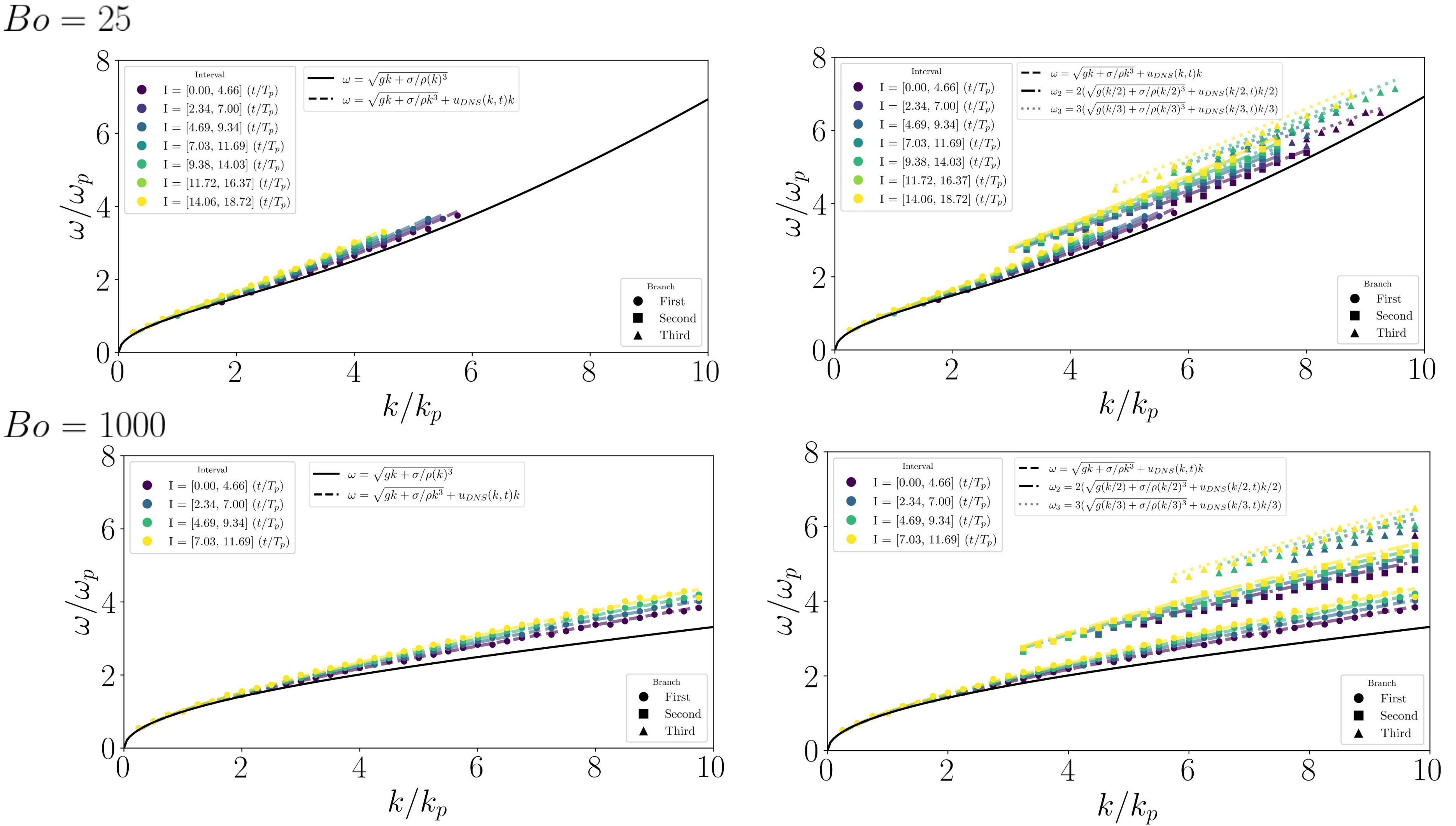
MECHANICAL & AEROSPACE ENGINEERING

High Meadows Environmental Institute

Stokes drift

$$u_{SD}(z) = 2g \int \frac{\phi(k)k^2}{\omega(k)} e^{2kz} dk$$





$Bo = 25$
$Bo = 1000$
ω/ω_p
k/k_p
Interval
I = [0.00, 4.66] (t/T_p)
I = [2.34, 7.00] (t/T_p)
I = [4.69, 9.34] (t/T_p)
I = [7.03, 11.69] (t/T_p)
I = [9.38, 14.03] (t/T_p)
I = [11.72, 16.37] (t/T_p)
I = [14.06, 18.72] (t/T_p)
$\omega = \sqrt{gk + \sigma/\rho(k)^3}$
$\omega = \sqrt{gk + \sigma/\rho k^3} + u_{DNS}(k,t)k$
$\omega_2 = 2(\sqrt{g(k/2) + \sigma/\rho(k/2)^3} + u_{DNS}(k/2,t)k/2)$
$\omega_3 = 3(\sqrt{g(k/3) + \sigma/\rho(k/3)^3} + u_{DNS}(k/3,t)k/3)$
Branch
First
Second
Third